[illegible]

$\log_n m = a \qquad \log_{mn} m^2 n = b \qquad a>0 \qquad [b]=?$ ۱-

$$\frac{\log_n m^2 n}{\log_n mn} = \frac{\log_n m + \log_n m + \log_n n}{\log_n m + \log_n n} = \frac{2a+1}{a+1} = b \qquad \frac{1}{a+1} - 2 = b$$

$$1 > \frac{1}{a+1} > 0 \to 2 > b > 1 \to [b] = 1$$

الف) $y=\sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}}$ ۲-

$\dfrac{x}{\log_{\frac{1}{2}} x} > 0 \qquad x>0 \qquad x \neq 1$

$x > \log_{\frac{1}{2}} x \qquad x < \sqrt{x} \to x^2 < x \to x^2 - x < 0 \quad x(x-1)<0$

$\boxed{0 < x < 1}$

ب) $y = \dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1} \to \log_4 (x^2-x-2) \geq 0 \to x^2-x-2 \geq 1 \to x^2-x-3 \geq 0$ ۳-

$x^2 \geq 1 \quad \left.\begin{matrix} x \geq 1 \\ x \leq -1 \end{matrix}\right\} (1)$

$\frac{1\pm\sqrt{13}}{2}$

$1 \cap 2 \to D_f = \left(-\infty, \frac{1-\sqrt{13}}{2}\right] \cup \left[\frac{1-\sqrt{13}}{2}, +\infty\right)$

$2\log_x a + \log_a \sqrt{x} = 2 \qquad \boxed{x=9} \qquad a=? \qquad \log_3 a = k$ ۳-

$2\log_9 a + \log_a 3 = 2$

$\hookrightarrow \log_3 a + \log_a 3 = 2 \qquad \dfrac{(\log_3 a)^2+1}{\log_3 a} = 2 \to (\log_3 a)^2 + 1 = 2\log_3 a$

$\log_a 3 \to \dfrac{1}{\log_3 a}$

$k^2 - 2k + 1 = 0 \qquad k=1$

$\log_3 a = 1 \to \boxed{a=3}$

$\log 2 \simeq ۰/۳ \qquad \log 3 = ۰/۴ \qquad (\log\frac{5}{3})x^2 + (\log 9)x - \log 15 = 0$ ۴-

$\log 5 = \log\frac{10}{2} = \cancel{\log 10} - \cancel{\log 2} = 1 - ۰/۳ = ۰/۷ \qquad (\log 5 - \overset{۰/۴}{\cancel{\log 3}})x^2 + (\overset{۰/۸}{\cancel{2\log 3}})x - (\log 5 + \overset{۰/۴}{\cancel{\log 3}})$

$۰/۳x^2 + ۰/۸x - ۱/۱ = 0 \qquad \boxed{x=1} \qquad \boxed{x=-\frac{11}{3}}$

$\log_2 7 = ۲/۸ \qquad \log_5 2 = ۰/۵ \qquad \log_{14} 15 = ?$ ۵

$\boxed{\log_2 5 = 2}$

$$\frac{\log_2 15}{\log_2 14} = \frac{\log_2 5 * \overset{1}{\cancel{\log_2 2}}}{\log_2 7 + \underset{1}{\cancel{\log_2 2}}} = \boxed{\frac{3}{۳/۸}}$$

$\log_3 5 = ۱/۵ \qquad \log_2 3 = ۱/۶ \qquad \log_{15} 6 = ?$ ۶-

$\frac{15}{10} = \frac{3}{2} \qquad \hookrightarrow \log_3 2 = \frac{10}{16} = \frac{5}{8}$

$$\frac{\log_3 6}{\log_3 15} = \frac{\overset{1}{\cancel{\log_3 3}} * \log_3 2}{\log_3 5 + \underset{1}{\cancel{\log_3 3}}} = \frac{\frac{13}{8}}{\frac{5}{2}} = \boxed{\frac{13}{20}}$$

$\log_8^{18} = m$ $\quad \log_4^{12} = ? \to 2\log_2^{12} \to 2(\log_2^{4} + \log_2^{3}) = 2 + \frac{3}{2}m - \frac{1}{2} = \boxed{\frac{3}{2}(1+m)}$ ۷-

$\log_8^9 + \cancel{\log_8^2} = m \quad \frac{2}{3}\log_2^3 = m - \frac{1}{3} \to \log_2^3 = \frac{3}{2}m - \frac{1}{2}$

$\frac{2}{3}\log_2^3 \quad \frac{1}{3}$

$(0/4)^{2x-1} = (\frac{125}{8})^{x^2} \quad \log_8^{(9x+1)} = ? \to \log_8(-8)$ ✗ ۸-

$\log_8^4 = \boxed{\frac{2}{3}}$

$(\frac{2}{5})^{2x-1} = (\frac{5}{2})^{3x^2} \quad -3x^2 = 2x - 1$

$3x^2 + 2x - 1 = 0 \to x = -1$ ✗ $\quad \frac{1}{3}$ ✓

$\log_2^3 = a \quad \log_8^b = \frac{2}{3}(1+a) \quad \log(3b-8) = ?$ ۹-

$2^a = 3 \quad b = 8^{\frac{2}{3}(1+a)} = (2^3)^{\frac{2}{3}(1+a)} = 2^{2(1+a)} = 2^{2+2a} = 2^2 \times 2^{2a} = 4\times 9 = 36$

$\log(\cancel{3b-8}) = \log 100 = \boxed{2}$

$-4ax^2 + bx + \frac{1}{2}c = 0 \quad \alpha + \beta = \log 4$ ۱۰-

$-\frac{b}{-4a} = \log 4 \quad \frac{1}{-\frac{b}{a}} = \log 4 \quad \frac{1}{-\frac{b}{4a}} = 2\log 2 \to \frac{2a}{b} = \log 2 \to b = \frac{2a}{\log 2} = 2a\log_2^{10}$

$2a = b + c \to 2a = 2a\log_2^{10} + c \to 2a(1 - \log_2^{10}) = c \to 2a(1 - (1 + \log_2^5)) = c$

$\log_2^2 + \log_2^5$

$c = -2a\log_2^5 \quad \frac{c}{a} = -2\log_2^5$

$(2^{-\frac{1}{8}})^{\frac{c}{a}} = (2^{-\frac{1}{8}})^{-2\log_2^5} = 2^{\frac{1}{4}\log_2^5}$

$5^{\frac{1}{4}\log_2^2} = \sqrt[4]{5}$