$\log_n m = a \qquad \log_{mn} m^2 n = b \qquad a>0 \qquad [b]=?$ صحیح

۱۸٫۵

۱- (۲)

$$\frac{\log_n m^2 n}{\log_n mn} = \frac{\log_n m+\log_n m+\log_n n}{\log_n m+\log_n n} = \frac{2a+1}{a+1} = b \qquad 2-\frac{1}{a+1} = b$$

$$b=1 \leftarrow 2>b>1 \leftarrow 1>\frac{1}{a+1}>0$$

۲- (۱٫۵)

دامنه $y=\sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}}$ $\qquad \dfrac{x}{\log_{\frac{1}{2}} x}>0 \qquad x>0 \qquad x\neq 1$

$x>\log_{\frac{1}{2}} x \qquad x<\sqrt{x} \to x^2<x \to x^2-x<0 \quad x(x-1)<0$

$\boxed{0<x<1}$

ب) $y=\dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1} \to \log \cancel{\dfrac{x^2-x-2}{2}} \geq 0 \to x^2-x-2 \geq 1 \to x^2-x-3\geq 0$

$x^2\geq 1 \quad \begin{cases} x\geq 1 \\ x<-1 \end{cases}$ (۱)

$۱۸٫۲ \to D_f = \left(-\infty, \frac{1-\sqrt{13}}{2}\right] \cup \left[\frac{1-\sqrt{13}}{2}, +\infty\right)$

$\frac{1+\sqrt{13}}{2} \qquad \frac{1-\sqrt{13}}{2}$

۳- (۲)

$2\log_x a + \log_a \sqrt{x} = 2 \qquad \boxed{x=9} \qquad a=? \qquad \log_3 a = k \qquad \log_3 a = 1 \to \boxed{a=3}$

$2\log_9 a + \log_a 3 = 2$

$\to \log_3 a + \log_a 3 = 2 \qquad \dfrac{(\log_3 a)^2+1}{\log_3 a} = 2 \to (\log_3 a)^2+1 = 2\log_3 a$

$\to \dfrac{1}{\log_3 a} \qquad k^2-2k+1=0 \qquad k=1$

۴- (۱٫۷۵)

$\log 2 \simeq 0.3 \qquad \log 3 = 0.4 \qquad (\log \frac{5}{3})x^2 + (\log 9)x - \log 15 = 0$

$\log 5 = \log\frac{10}{2} = \cancel{\log 10} - \cancel{\log 2} = 1-0.3 = 0.7 \qquad (\log 5 - \overset{0.4}{\cancel{\log 3}})x^2 + (\overset{0.8}{\cancel{2\log 3}})x - (\log 5 + \overset{0.4}{\cancel{\log 3}})$

$0.3x^2+0.8x-1.1=0 \qquad \boxed{x=1} \qquad \boxed{x=-\frac{11}{3}}$

اختلاف $\rightarrow$ $\boxed{\frac{14}{3}}$

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۵ (۲)

$\log_2 7 = 2.8 \qquad \log_5 2 = 0.5 \qquad \log_{14} 10 = ?$

$\boxed{\log_2 5 = 2}$

$$\frac{\log_2 10}{\log_2 14} = \frac{\log_2 5 + \overset{1}{\cancel{\log_2 2}}}{\log_2 7 + \underset{1}{\cancel{\log_2 2}}} = \boxed{\frac{3}{3.8}}$$

۶- (۲)

$\log_3 5 = 1.5 \qquad \log_2 3 = 1.6 \qquad \log_{15} 6 = ?$

$\frac{15}{10}=\frac{3}{2} \qquad \to \log_3 2 = \frac{10}{16} = \frac{5}{8}$

$$\frac{\log_3 6}{\log_3 15} = \frac{\overset{1}{\cancel{\log_3 3}}+\log_3 2}{\log_3 5 + \underset{1}{\cancel{\log_3 3}}} = \frac{\frac{13}{8}}{\frac{5}{2}} = \boxed{\frac{13}{20}}$$

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$\log_8 18 = m \qquad \log_4 12 = ? \rightarrow \frac{1}{2}\log_2 12 \rightarrow \frac{1}{2}(\log_2 4 + \log_2 3) = 2 + \frac{3}{2}m - \frac{1}{2} = \boxed{\frac{3}{2}(1+m)}$ — ۷ (۱,۷۵)

!دقت

$\log_8 9 + \log_8 2 = m$

$\frac{2}{3}\log_2 3 + \frac{1}{3}$

$\frac{2}{3}\log_2 3 = m - \frac{1}{3} \rightarrow \log_2 3 = \frac{3}{2}m - \frac{1}{2}$

$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \qquad \log_8 (9x+1) = ?$ — ۸ (۱,۵)

$\left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2}$

$-3x^2 = 2x - 1$

$3x^2 + 2x - 1 = 0 \rightarrow x = -1$ ✗ $,\ \frac{1}{3}$ ✓

$\log_8(-8)$ ✗ $\qquad \log_8 4 = \boxed{\cancel{1}}$

$\lg_{2^3} 2^2 = \frac{2}{3}\lg_2 2 = \frac{2}{3}$

$\log_2 3 = a \qquad \log_8 b = \frac{2}{3}(1+a) \qquad \log(3b-8) = ?$ — ۹ (۲)

$2^a = 3 \qquad b = 8^{\frac{2}{3}(1+a)} = (2^3)^{\frac{2}{3}(1+a)} = 2^{2(1+a)} = 2^{2+2a} = 2^2 \times 2^{2a} = 4 \times 9 = 36$

$\log 100 = \log\left(\cancel{3b-8}\right) = \boxed{2}$

$-4ax^2 + bx + \frac{1}{2}c = 0 \qquad \alpha + \beta = \log 4$ — ۱۰ (۲)

$-\frac{b}{-4a} = \log 4 \qquad \frac{1}{\frac{b}{4a}} = \log 4 \qquad \frac{1}{\frac{b}{4a}} = 2\log 2 \rightarrow \frac{2a}{b} = \log 2 \rightarrow b = \frac{2a}{\log 2} = 2a\log_2 b$

$2a = b + c \rightarrow 2a = 2a\log_2 10 + c \rightarrow 2a(1 - \log_2 10) = c \rightarrow 2a(1 - (1 + \log_2 5)) = c$

$\log_2 2 + \log_2 5$

$c = -2a\log_2 5 \qquad \frac{c}{a} = -2\log_2 5$

$\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = \left(2^{-\frac{1}{8}}\right)^{-2\log_2 5} = 2^{\frac{1}{4}\log_2 5}$

$5^{\frac{1}{4}\log_2 2} = \sqrt[4]{5}$

۲-

ب) $x^2 - x - 2 > 0 \rightarrow (-\infty, -1) \cup (2, +\infty)$ (۱)

$x^2 - 1 \geq 0 \rightarrow x \geq 1,\ x \leq -1$ (۲)

$\xrightarrow{\cap} (-\infty, -1) \cup (2, +\infty)$