سهیل بهرامی          تکالیف ۲۳          یازدهم بهر B

۱)

$\log_n m = a \qquad \log_{mn} m^2 n = b \qquad \log_{mn} mn\times n = b \Rightarrow 1+\log_{mn} n = b \Rightarrow 1+\frac{1}{\log_n mn} = b \Rightarrow 1+\frac{1}{1+\log_n m} = b$

$\Rightarrow 1+\frac{1}{1+a} = b \Rightarrow \frac{2a+1}{a+1} = b \qquad \frac{a+1}{a+1} < b < \frac{2a+2}{a+1} \Rightarrow 1 < b < 2 \Rightarrow [b] = 1$

---

۲)

الف) $y = \sqrt{\frac{x}{\log_{\frac12} x}} \qquad \log_{\frac12} x = 0 \Rightarrow x = (\frac12)^0 = 1$

| $x$ | | $0$ | | $1$ | |
|---|---|---|---|---|---|
| $y$ | $-$ | | $+$ | | $-$ |

$\log_{\frac12} x \to x > 0$

$D_f = (0,1)$

ب) $y = \frac{\log_e (x^2-x-2)}{\sqrt{x^2-1}+1} \to \log_e (x^2-x-2) \to x^2-x-2 > 0 \Rightarrow (x-2)(x+1) > 0$

| $x$ | | $-1$ | | $2$ | |
|---|---|---|---|---|---|
| $y$ | $+$ | | $-$ | | $+$ |

$\sqrt{x^2-1} \to x^2 \geq 1 \Rightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases} \qquad D_f = (-\infty,-1)\cup(2,+\infty \qquad (-\infty,1)\cup(2,+\infty)$

---

۳)

$2\log_x a + \log_a \sqrt{x} = 2 \qquad \log_x a^2 + \frac12 \log_a x = 2 \Rightarrow \begin{cases} a > 0 \\ a \neq 1 \end{cases}$

$\log_x a^2 + \log_{a^2} x = 2 \Rightarrow \log_x a^2 + \frac{1}{\log_x a^2} = 2 \Rightarrow t + \frac1t = 2 \Rightarrow t^2 - 2t + 1 = 0 \to (t-1)^2 = 0 \Rightarrow t = 1$

$\log_x a^2 = 1 \xRightarrow{x=3} \log_9 a^2 = 1 \Rightarrow a^2 = 9 \Rightarrow a = 3$ ق ق

$a = -3$ غ ق ق

---

٤)

$\log 2 = 0.3 \qquad \log 3 = 0.4 \qquad (\log \frac52)x^2 + (\log 9)x - \underbrace{\log 15}_{\log 3 + \log 5} = 0 \qquad 1 - \log 2 = \log 5 \Rightarrow \log 5 = 0.7$

$(\log 5 - \log 2)x^2 + (2\log 3)x - (\log 5 + \log 3) = 0$

$\Rightarrow 0.3x^2 + 0.8x - 1.1 = 0$

$3x^2 + 8x - 11 = 0 \Rightarrow \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{64+132}}{3} = \frac{14}{3}$

---

۵)

$\log_2 7 = 2.8 \qquad \log_5 2 = 0.5 \qquad \log_{14} 10 = ?$

$\log_5 7 = \frac{\log_2 7}{\log_2 5} = \frac{2.8}{2} = 1.4 \qquad \log_{14} 10 = \log_{14} 5 + \log_{14} 2 = \frac{1}{\log_5 14} + \frac{1}{\log_2 14} = \frac{1}{\log_5 7 + \log_5 2} + \frac{1}{1+\log_2 7}$

$\frac{1}{1.4+0.5} + \frac{1}{1+2.8} = \frac{1}{1.9} + \frac{1}{3.8} = \frac{3}{3.8} = \frac{15}{19}$

۱۲

$\log_3 5 = 1.5 \qquad \log_2 3 = 1.6 \qquad \log_{15} 6 = ?$

$\log_3 2 = \frac{1}{1.6}$

$\log_2 5 = \frac{\log_3 5}{\log_3 2} = \frac{1.5}{\frac{1}{1.6}} = \frac{12}{5} = 2.4$

$\log_{15} 6 = \log_{15} 2 + \log_{15} 3 = \frac{1}{\log_2 15} + \frac{1}{\log_3 15} = \frac{1}{\log_2 3 + \log_2 5} + \frac{1}{\log_3 5 + \log_3 3} = \frac{1}{1.6+2.4} + \frac{1}{1.5+1} = \frac{1}{4} + \frac{1}{2.5}$

$= 0.65$

---

۱۳

$\log_8 18 = m \qquad \log_4 12$

$\frac{1}{3}\log_2 18 = \frac{1}{3}\left(\log_2 (2\times 3\times 3)\right) = \frac{1}{3}\left(1 + 2\log_2 3\right) = m$

$\Rightarrow 3m = 1 + 2\log_2 3 \Rightarrow \frac{3m-1}{2} = \log_2 3 \Rightarrow \log_4 12 = \frac{1}{2}\log_2 12 = \frac{1}{2}\left(2 + \log_2 3\right) = \frac{1}{2}\left(\frac{3m-1+4}{2}\right) = \frac{3m+3}{4}$

$\frac{3}{4}(m+1)$

---

۱۴

$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \qquad \log_8 (9x+1)$

$\left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2}$

$\left(\frac{5}{2}\right)^{-2x+1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow 3x^2 = -2x+1 \Rightarrow 3x^2 + 2x - 1 = 0 \Rightarrow \frac{(x+3)(x-1)}{3}$

$x_1 = -\frac{3}{3} = -1$ غ ق

$x_2 = \frac{1}{3}$ ق ق

$\log_8 (9x+1) \Rightarrow 9x+1 > 0 \Rightarrow x > -\frac{1}{9} \qquad \log_8 \left(\frac{9}{3}+1\right) = \log_8 4 = \frac{2}{3}\log_2 2 = \frac{2}{3}$

---

۹

$\log_2 3 = a \qquad \log_8 b = \frac{2}{3}(1+a)$

$\log_8 b = \frac{2}{3}\left(1 + \log_2 3\right) \Rightarrow \frac{1}{3}\log_2 b = \frac{2}{3}\log_2 6 \Rightarrow \log_2 b = \log_2 36$

$b = 36$

$\log 3b-8 = \log (3\times 36)-8 = \log 100 = 2$

---

۱۰

$-4ax^2 + bx + \frac{1}{2}c = 0 \qquad \frac{1}{\alpha+\beta} = \log 4 \qquad a = \frac{b+c}{2} \to b = 2a-c$

$\frac{1}{\alpha+\beta} = \log 4 \Rightarrow \frac{1}{S} = \frac{4a}{b} = \frac{\cancel{4}^{2}a}{2a-c} = \log 4 = \cancel{2}\log 2 \to \frac{2a}{2a-c} = \log 2$

$\to \frac{2a-c}{2a} = \log_2 10 \to 1 - \log_2 10 = \frac{c}{2a} \Rightarrow 2 - \log_2 100 = \frac{c}{a}$

$\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = \left(2^{\frac{c}{a}}\right)^{-\frac{1}{8}} = \left(2^{2-\log_2 100}\right)^{-\frac{1}{8}} = \left(\frac{4}{100}\right)^{-\frac{1}{8}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$