امیرمهدی نزاری کلاس یازده تجربی B ۲۳

①

$$\log_{mn} m^2n = \log_{mn} m + 1$$

$$\log_{mn} m = \frac{1}{\log_m mn} = \frac{1}{1+\frac{1}{a}} = \frac{a}{a+1}$$

$$\Rightarrow \log_{mn} m^2n = \frac{2a+1}{a+1} = b$$

$$\Rightarrow 1+\frac{a}{a+1} = b \Rightarrow \left[1+\frac{a}{a+1}\right] = 1 + \cancel{\left[\frac{a}{a+1}\right]}^{\,0} = 1 = [b]$$

$$a < a+1$$

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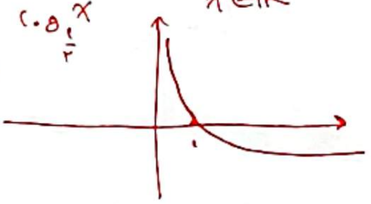
② الف)

$$\log_{\frac{1}{2}} x \Rightarrow x > 0 \quad x \in \mathbb{R}^+ \qquad \frac{x}{\log_{\frac{1}{2}} x} \geq 0 \overset{x>0}{\Rightarrow} \log_{\frac{1}{2}} x > 0$$

$\Rightarrow$ $\log_{\frac{1}{2}} x$



$$\Rightarrow x \in (0,1) \cap \mathbb{R}^+ \Rightarrow D = (0,1)$$

ب)

$$\log_4 (x^2 - x - 2) \Rightarrow x^2 - x - 2 > 0 \Rightarrow + \overset{-1}{\phi} - \overset{2}{\phi} +$$

$$\Rightarrow x \in \mathbb{R} - [-1,2] \ ①$$

$$x^2 - 1 \geq 0 \Rightarrow + \overset{-1}{\phi} - \overset{1}{\phi} + \Rightarrow x \in \mathbb{R} - (-1,1) \ ②$$

$$① \cap ② = \mathbb{R} - [-1,2] = D$$

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③

$$2\log_x a = \log_{\sqrt{x}} a$$

$$\log_a \sqrt{x} + \log_{\sqrt{x}} a = \log_a \sqrt{x} + \frac{1}{\log_a \sqrt{x}} = 2 \Rightarrow t + \frac{1}{t} = 2$$

$$\Rightarrow t^2 - 2t + 1 = 0 \Rightarrow t = 1 \Rightarrow \log_a \sqrt{x} = 1 \Rightarrow a = \sqrt{x} \overset{x=9}{\Rightarrow} \sqrt{9} = a$$

$$\Rightarrow a = 3$$

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④

$$\log 2 \simeq 0.3 \Rightarrow \log 5 \simeq 0.7$$

$$\log \frac{5}{4} = \log 5 - \log 4 = 0.7 - 0.4 = 0.3$$

$$\log 9 = 0.8 , \quad \log 15 = \log 5 + \log 3 = 1.1$$

$$\Rightarrow 0.3x^2 + 0.8x - 1.1 = 0 \Rightarrow 3x^2 + 8x - 11 = 0$$

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{196}}{3}$$

$\log_{۲}^{۷} = ۲٫۸ \Rightarrow \frac{\log_{۲}^{۷}}{\log_{۲}^{۵}} = \boxed{۱٫۴ = \log_{۵}^{۷}}$

$\log_{۵}^{۲} = ۰٫۵$

$\log_{۱۴}^{۲} + \log_{۱۴}^{۵} = \frac{1}{\log_{۲}^{۷} + \log_{۲}^{۲}} + \frac{1}{\log_{۵}^{۷} + \log_{۵}^{۲}} = \frac{1}{۲٫۸+۱} + \frac{1}{۱٫۴+۰٫۵}$

$\frac{۱۰}{۳۸} + \frac{۱۰}{۱۹} = \frac{۵۷۰}{۷۲۲}$

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(۶)

$\log_{۳}^{۵} = \frac{\log_{۲}^{۵}}{\log_{۲}^{۳}} = \frac{۱٫۵}{۱٫۲} = ۲٫۴$

$\log_{۱۵}^{۳} + \log_{۱۵}^{۲} = \frac{1}{\log_{۳}^{۱۵}} + \frac{1}{\log_{۲}^{۱۵}} = \frac{1}{\log_{۳}^{۵} + \log_{۳}^{۳}} + \frac{1}{\log_{۲}^{۵} + \log_{۲}^{۳}}$

$= \frac{1}{۱+۱٫۵} + \frac{1}{۲٫۴+۱٫۶} = \frac{۱۰}{۲۵} + \frac{۱}{۴} = \frac{۶۵}{۱۰۰}$

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(۷)

$\log_{۸}^{۹} + 1 = m \Rightarrow \frac{۲}{۳}\log_{۲}^{۳} = m - 1 \Rightarrow \log_{۲}^{۳} = \frac{۳}{۲}(m-1)$

$\log_{۴}^{۴} + \log_{۴}^{۳} = 1 + \frac{\log_{۲}^{۳}}{۲} = 1 + \frac{۳}{۴}(m-1) = \frac{۳m+1}{۴}$

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(۸)

$\frac{۲}{۵}^{۲x-1} = \left(\frac{۵}{۲}\right)^{۳x^۲} \Rightarrow ۳x^۲ = -۲x+1$

$۳x^۲ + ۲x - 1 = 0 \quad \frac{-۲ \pm ۴}{۶} = \frac{1}{۳}, -1 \Rightarrow \log_{۸}^{۹x+1} = \log_{۸}^{۴} =$

$\frac{۲}{۳}$

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(۹)

$1 + a = 1 + \log_{۲}^{۳} = \log_{۲}^{۲} + \log_{۲}^{۳} = \log_{۲}^{۶}$

$\frac{۲}{۳}\log_{۲}^{۶} = \log_{۸}^{۳۶} = \log_{۸}^{b} \Rightarrow \boxed{b = ۳۶} \Rightarrow \log(۱۰۸ - ۸) = \log ۱۰۰ = ۲$

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(۱۰)

$\log^{۴} = -\frac{b}{۲c} \Rightarrow \frac{b}{c} = \frac{1}{\log_{۱۰}^{۱۶}} \Rightarrow \frac{b}{c} = \log_{۱۶}^{۱۰}$

$\frac{b+c}{۲} = a \Rightarrow b + c = ۲a \Rightarrow b = c - ۲a \Rightarrow \frac{b}{c} = 1 - \frac{۲a}{c}$

$-\frac{۲a}{c} = \log_{۱۶}^{\frac{۵}{۸}} \Rightarrow \frac{a}{c} = \log_{\frac{۵}{۸}}^{۲۵۶} \Rightarrow \frac{c}{a} = \log_{۲۵۶}^{\frac{۵}{۸}}$

$\Rightarrow \frac{c}{a} = \frac{1}{۸} \times (\log_{۲}^{۵} - ۳) \Rightarrow \left(\left(۲^{\frac{1}{۸}}\right)^{\frac{1}{۸}}\right)^{\log_{۲}^{۵} - ۳} = \frac{۵^{\frac{1}{۶۴}}}{۸^{\frac{1}{۶۴}}} = \frac{۸^{\frac{1}{۶۴}}}{۵^{\frac{1}{۶۴}}}$

$= \sqrt[۶۴]{\frac{۸}{۵}}$