

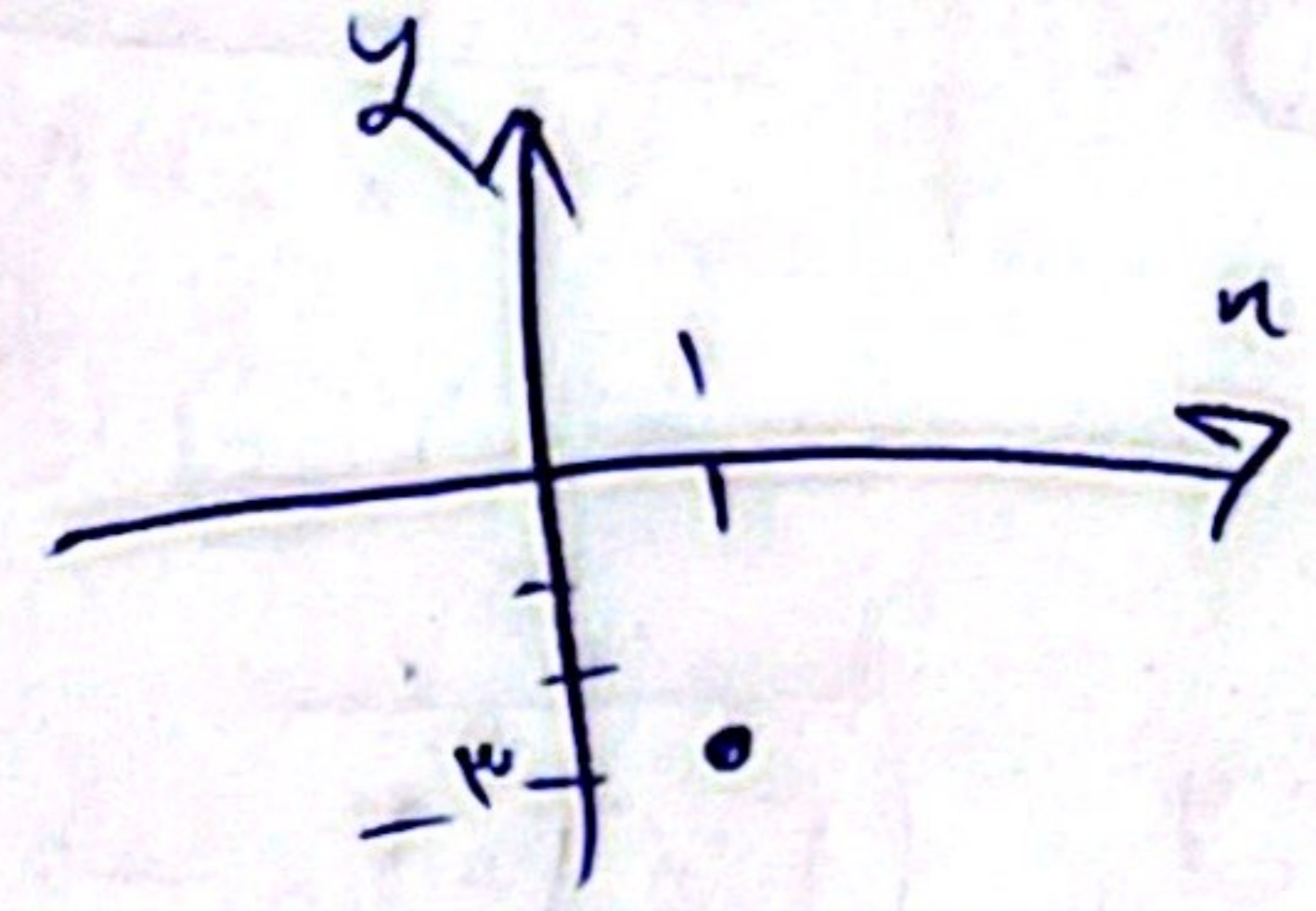
$$2x^2 + y^2 - 4x + 4y + k = 0$$

$$(y+2)^2 - 4 + 2(x-1)^2 - 2 + k = 0$$

$$(y+2)^2 + 2(x-1)^2 = 11 - k$$

$$\frac{(y+2)^2}{2} \geq 0 \rightarrow 11 - k = 0 \rightarrow 11 = k \quad \checkmark$$

تابع مرکز نقطه $(1, -2)$



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$$f(a) = a^2 + (a-2)(a+2)$$

$$a^2 + a - 2 \rightarrow a=1 \rightarrow a=2$$

$$(a+2)(a-1)$$

$$a=1 \Rightarrow f(n) = \begin{cases} n^2 + 6n & ; n \geq 1 \\ 2n + 3 & ; n \leq 1 \end{cases}$$

$$f(1) = 1 + 6 = 7 \quad \checkmark$$

$$f(n) = \begin{cases} n^2 + 6n & ; n \geq 1 \\ 2n + 3 & ; n \leq 1 \end{cases}$$

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$$f(n) = \begin{cases} 2n^2 - b & ; |n+1| \geq 2 \rightarrow \begin{matrix} n+1 \geq 2 \rightarrow n \geq 1 \\ n+1 \leq -2 \rightarrow n \leq -3 \end{matrix} \\ a + 2n & ; -3 \leq n \leq -1 \end{cases}$$

$$f(-3) = 18 - b = a - 4$$

$$4b = a + b \quad \checkmark$$

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$$g = \sqrt{4x-a} + \sqrt{b-3x}$$

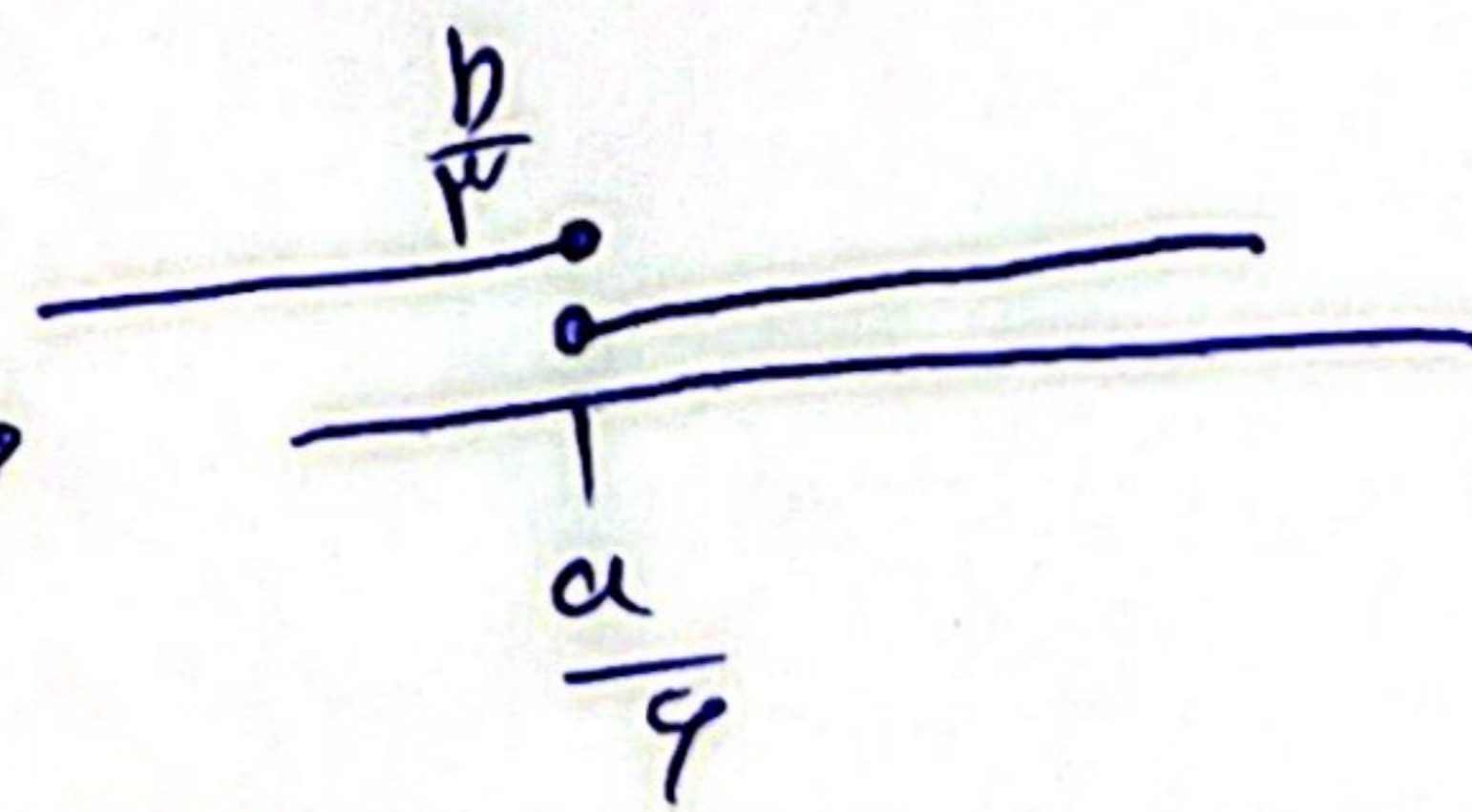
$$D_f = \{x\}$$

$$4x - a \geq 0$$

$$x \geq \frac{a}{4} \quad \text{I}$$

$$b - 3x \geq 0$$

$$\frac{b}{3} \geq x \quad \text{II}$$



$$\frac{b}{3} \geq \frac{a}{4} \Rightarrow b \geq \frac{3a}{4}$$

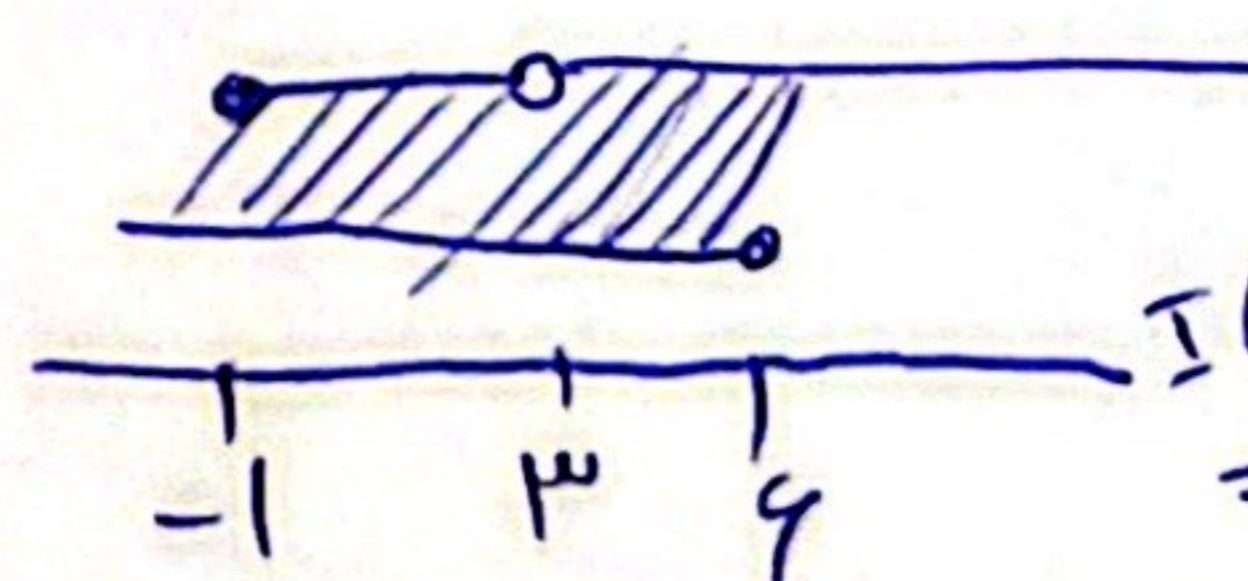
$$\frac{b}{a} \geq \frac{3}{4} \Rightarrow \frac{b}{a} \geq \frac{3}{4} \quad \checkmark$$

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$$g(x) = \sqrt{\frac{x+1}{1-x-1}} + \sqrt{\frac{4-x}{x^2+2x+1}}$$

$$\frac{-1}{-1-x-1} \Rightarrow \frac{1}{-x-2}$$

$$\frac{4}{x^2+2x+1} \Rightarrow \frac{4}{(x+1)^2}$$



$$D_f = [-1, 4] - \{3\}$$

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$$g(x) = \sqrt{[x]-2} + \sqrt{4-[x]}$$

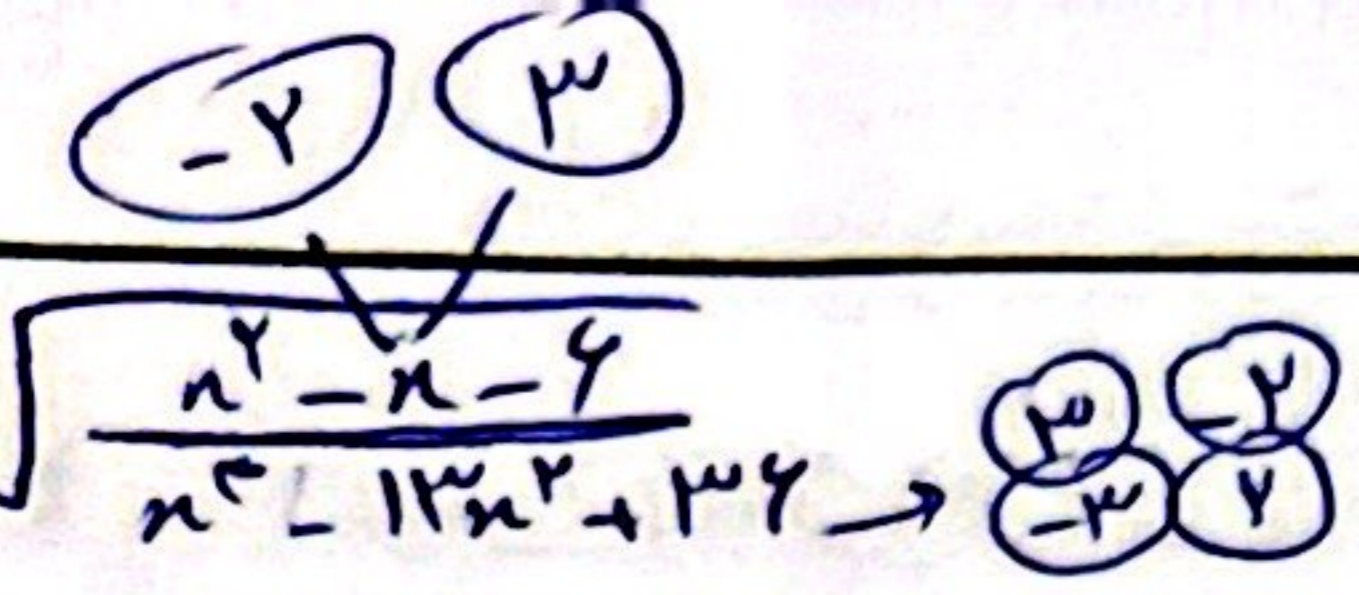
$$[x]-2 \geq 0 \rightarrow [x] \geq 2 \rightarrow x \geq 2 \quad \text{I}$$

$$4-[x] \geq 0$$

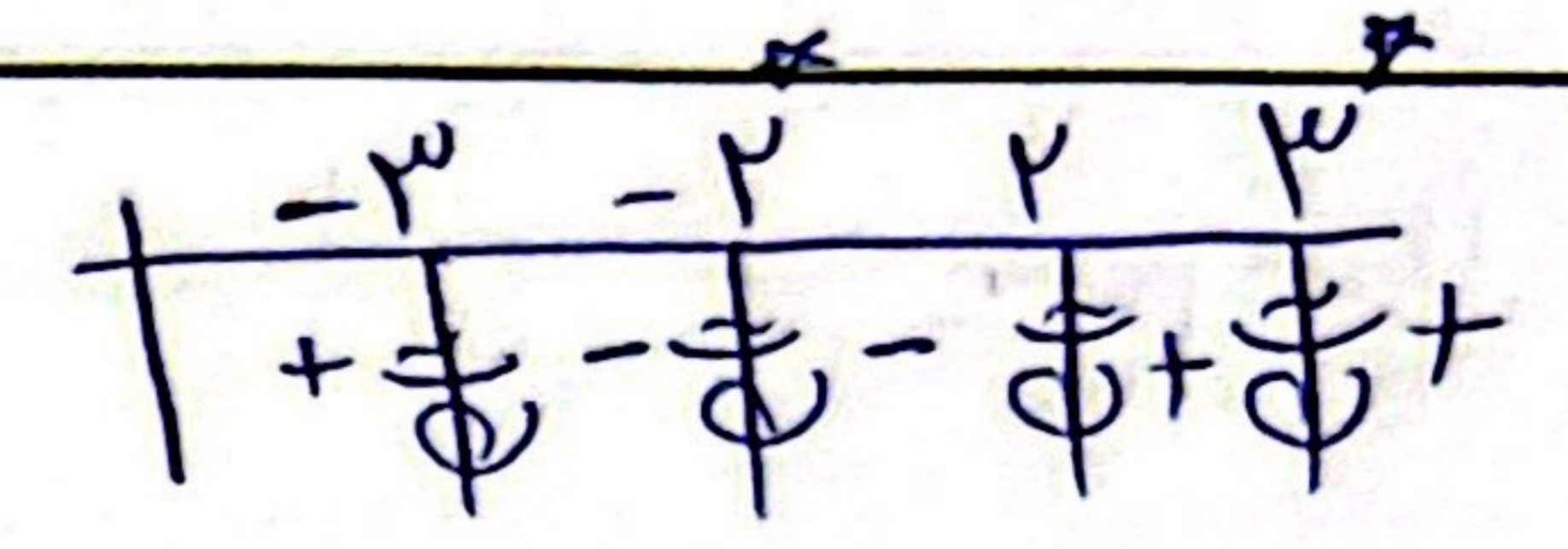
$$[x] \leq 4 \rightarrow x < 5 \quad \text{II}$$

$$\text{I} \cap \text{II} \Rightarrow D_f = \emptyset \quad \checkmark$$

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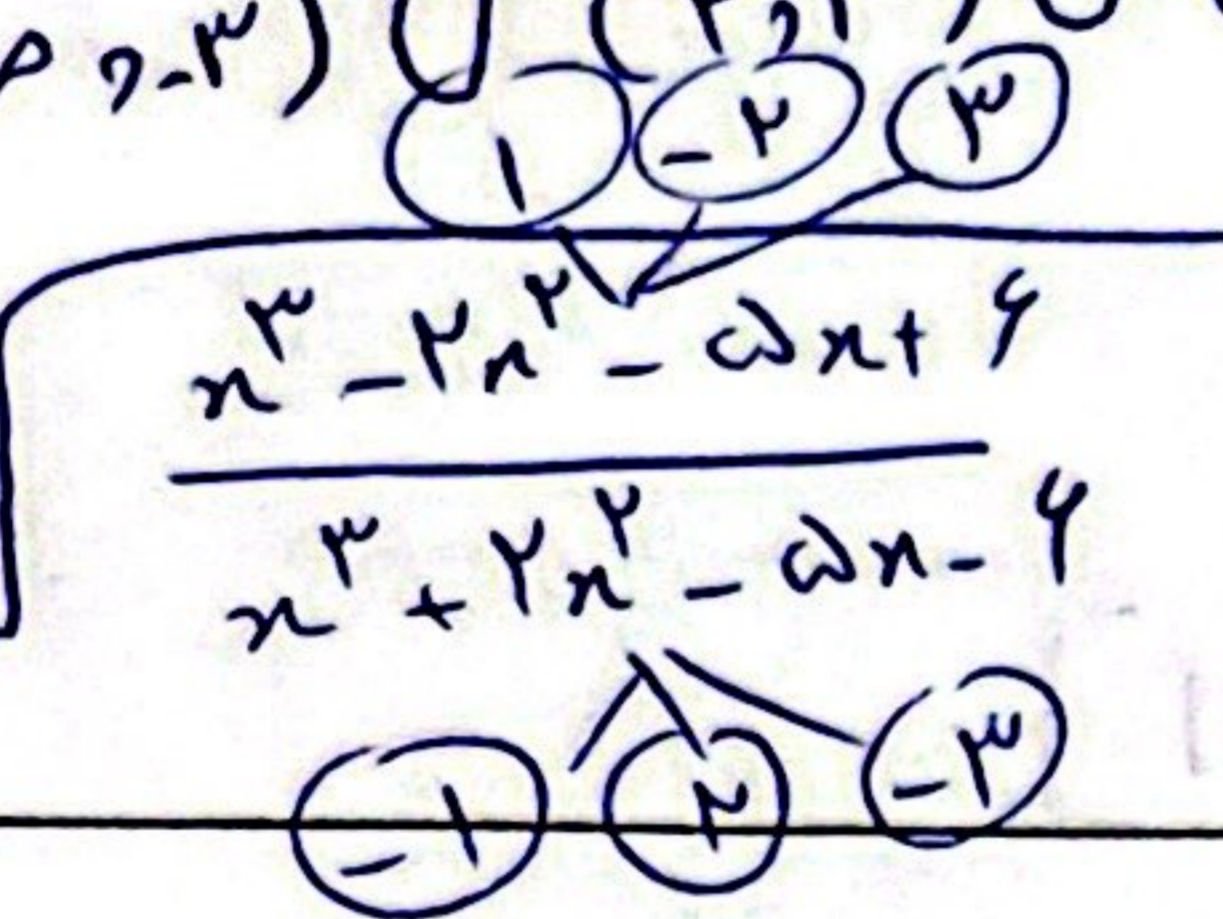
الف) $y = \sqrt{\frac{x^2 - x - 4}{x^2 - 13x^2 + 34}} \rightarrow$ 

$x^2 - x - 4 = (x - 2)(x + 2)$
 $x^2 - 13x^2 + 34 = (x^2 - 9)(x^2 - 4) = (x - 3)(x + 3)(x - 2)(x + 2)$

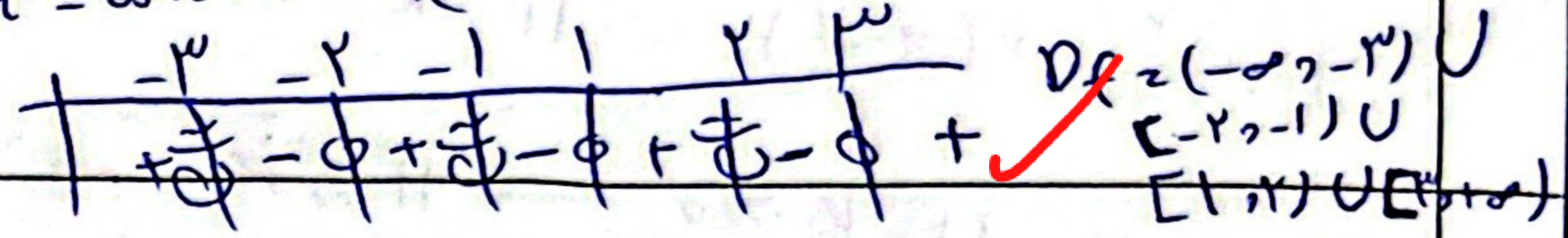


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$D_f = (-\infty, -3) \cup (-2, 2) \cup (2, +\infty)$

ب) $y = \sqrt{\frac{x^3 - 2x^2 - 2x + 4}{x^3 + 2x^2 - 2x - 4}}$ 

$x^3 - 2x^2 - 2x + 4 = (x - 1)(x + 2)(x - 2)$
 $x^3 + 2x^2 - 2x - 4 = (x + 1)(x - 2)(x + 2)$



$D_f = (-\infty, -3) \cup (-2, -1) \cup (1, 2) \cup (2, +\infty)$

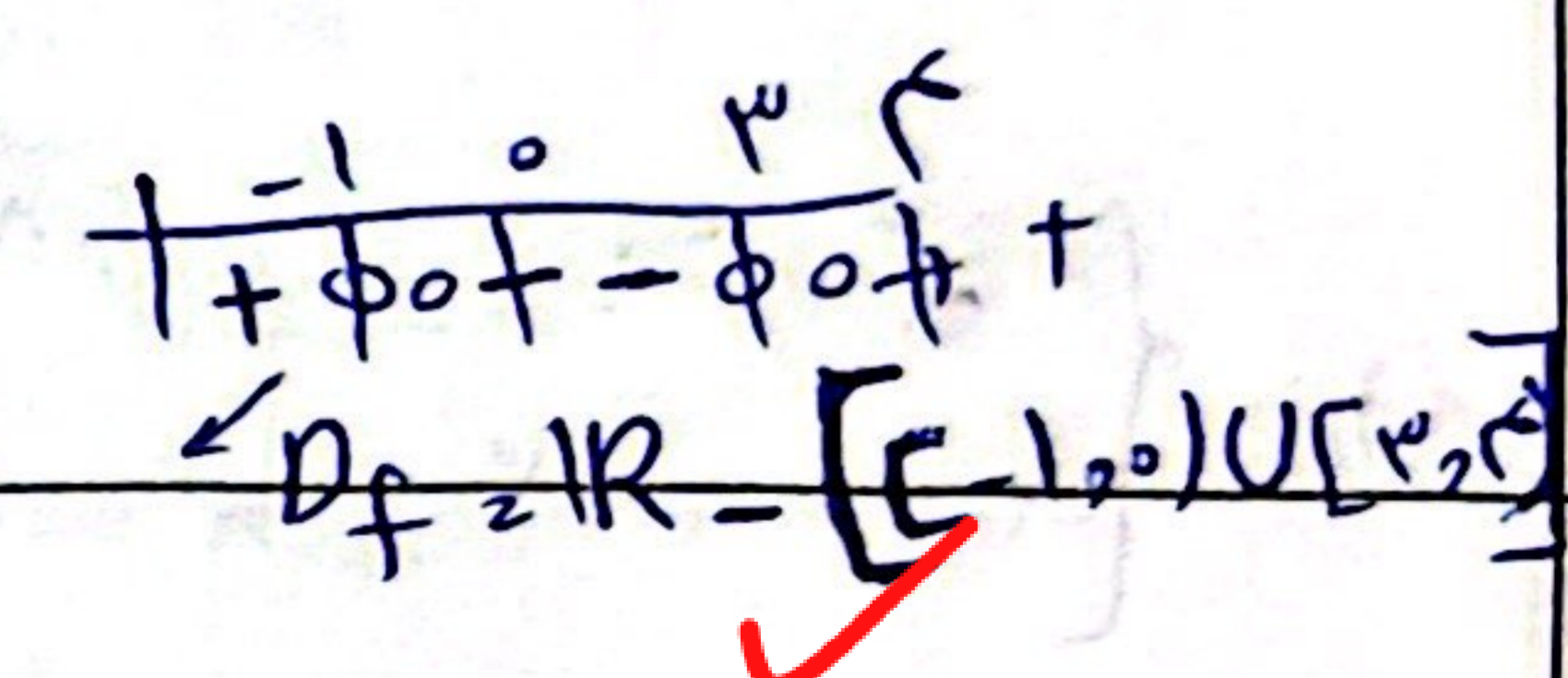
الف) $y = \sqrt{[-x] - 2} \rightarrow [-x] - 2 \geq 0$

$[-x] \geq 2 \rightarrow -x \geq 2$
 $\rightarrow x \leq -2 \rightarrow D_f = (-\infty, -2]$

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ب) $y = \frac{\omega x^2 + 4}{[x]^2 - 2[x] - 3} \rightarrow [x]^2 - 2[x] - 3 \neq 0$

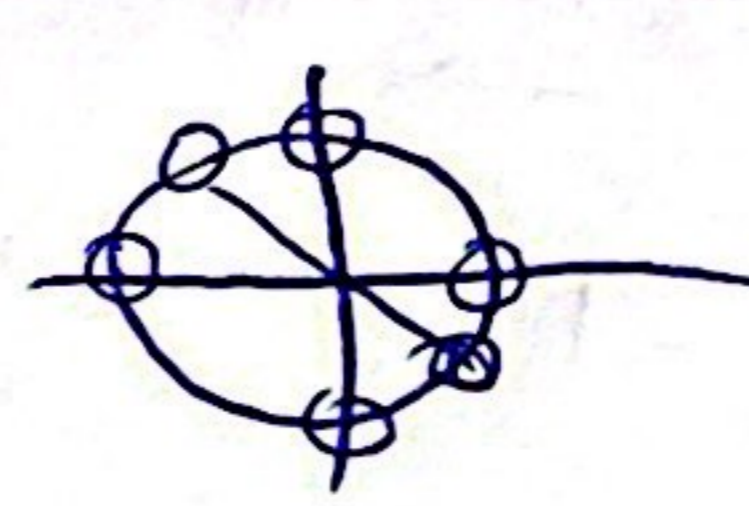
$([x] - 3)([x] + 1) \neq 0$
 $[x] \in (-1, 3)$



$D_f = \mathbb{R} - [-1, 3]$

الف) $y = \frac{\cot x + 1}{\tan x + 1} = \frac{\frac{\cos x}{\sin x} + 1}{\frac{\sin x}{\cos x} + 1}$

$\cos x, \sin x \neq 0, \tan x \neq -1$

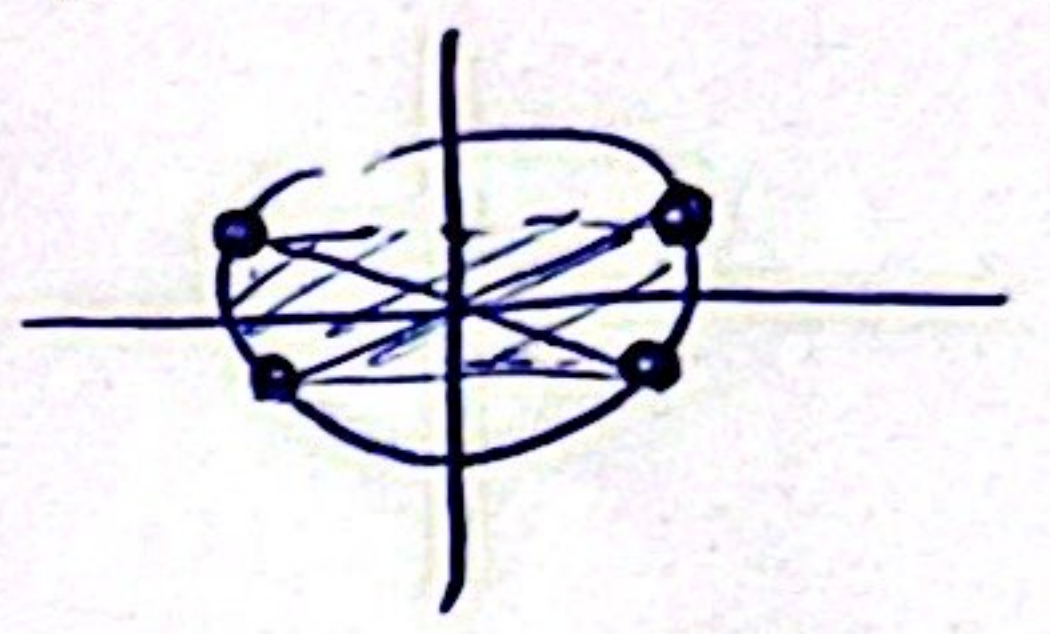


$D_f = \mathbb{R} - \left\{ \frac{k\pi}{2}, k\pi - \frac{\pi}{4} \right\}$

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$\wedge) \text{ ب) } y = \sqrt{1 - \epsilon \sin^2 x} \rightarrow 1 - \epsilon \sin^2 x \geq 0$
 $\frac{1}{\epsilon} \geq \sin^2 x \rightarrow -\frac{1}{\sqrt{\epsilon}} \leq \sin x \leq \frac{1}{\sqrt{\epsilon}}$

$D_f = \left[k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4} \right] \checkmark$



$9) \text{ الف) } y = \sqrt{1 - \log_{\frac{1}{\epsilon}} x^{-1}} \rightarrow 1 - \log_{\frac{1}{\epsilon}} x^{-1} \geq 0$
 $1 \geq \log_{\frac{1}{\epsilon}} x^{-1}$
 $\frac{1}{\epsilon} \leq x^{-1} \rightarrow \frac{\epsilon}{1} \leq x \rightarrow D_f = \left[\frac{\epsilon}{1}, +\infty \right) \checkmark$

$9) \text{ ب) } \frac{\sqrt{x^2 - n}}{1 - \log_{\frac{1}{\epsilon}} x^{-1}}$
 $x^2 - n > 0$

$\textcircled{1} \quad x^2 - n \geq 0 \rightarrow \frac{1}{\sqrt{x^2 - n}}$
 $\textcircled{2} \quad 1 - \log_{\frac{1}{\epsilon}} x^{-1} \neq 0 \rightarrow \epsilon + \frac{n^2 - 1}{x} \neq 0$
 $x^2 - n - \epsilon \neq 0$
 $x \neq \sqrt{n}$
 $x \neq -1$

$\textcircled{2}$

$\frac{1}{\sqrt{x^2 - n}} \rightarrow \textcircled{3}$

$D_f = (-\infty, 0) \cup (\sqrt{n}, +\infty) - \{-1, \sqrt{n}\} \checkmark$

$10) \text{ الف) } y = \sqrt{\frac{\epsilon x - x^2}{\epsilon} - 1}$
 $\frac{\epsilon x - x^2}{\epsilon} - 1 \geq 0$

$\frac{\epsilon x - x^2}{\epsilon} \geq 1$
 $\epsilon x - x^2 \geq \epsilon$
 $x^2 - \epsilon x + \epsilon \leq 0$
 $(x - \frac{\epsilon}{2})^2 - \frac{\epsilon^2}{4} + \epsilon \leq 0$
 $(x - \frac{\epsilon}{2})^2 \leq \frac{\epsilon^2}{4} - \epsilon$

$D_f = [1, \epsilon] \checkmark$

$\textcircled{2}$

$10) \text{ ب) } y = \left(\frac{\epsilon x + \omega}{\epsilon x + \epsilon} \right) ! \rightarrow \frac{\epsilon x + \omega}{\epsilon x + \epsilon} \in \mathbb{W}$

$D_f = \left\{ x \mid x = \frac{\epsilon k + \omega}{\epsilon k - \epsilon}, k \in \mathbb{W} \right\} \checkmark$