



$$\rightarrow H = \left(\frac{3+1}{2}, \frac{1-4}{2} \right) = H(2, -1.5)$$

$$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{1-(-4)}{3-1} = \frac{5}{2} = 2.5 \Rightarrow m_{CH} = -\frac{1}{2.5} = -\frac{2}{5}$$

$$\Rightarrow y - 1 = \frac{1}{5}(x - 3) \rightarrow y = \frac{x}{5} + \frac{4}{5}$$

$$\text{طول } BC = \sqrt{14+9} = 5 \quad m_{BC} = \frac{\Delta y}{\Delta x} = \frac{5}{-3} = -\frac{5}{3} \quad \overline{BC} \rightarrow y = -\frac{5}{3}x + \frac{5}{3}$$

$$\rightarrow m_{AH} = \frac{3}{5} \quad \overline{AH} \rightarrow y - 1 = \frac{3}{5}(x - 3) \rightarrow y = \frac{3}{5}x - \frac{4}{5}$$

$$\text{نقطه } H: \frac{3}{5}x - \frac{4}{5} = -\frac{5}{3}x + \frac{5}{3} \rightarrow x = \frac{17}{8} \quad \text{نقطه } H: y = -\frac{1}{5} \Rightarrow H\left(\frac{17}{8}, -\frac{1}{5}\right)$$

$$\text{طول } AH = \sqrt{\left(\frac{17}{8}\right)^2 + \left(-\frac{1}{5}\right)^2} = \sqrt{\frac{289}{64} + \frac{1}{25}} = \sqrt{\frac{7225 + 64}{1600}} = \sqrt{\frac{7289}{1600}} = \frac{\sqrt{7289}}{40}$$

$$\text{فاصله دایره} = \text{فاصله نقطه} \Rightarrow m_1 = m_2 \Rightarrow \frac{1}{2} = \frac{a}{3} \Rightarrow a = 1.5$$

$$y = 2x + 2$$

$$y = x + 7 \rightarrow y = 2x + 14$$

$$\text{فاصله نقطه} = \frac{14-2}{\sqrt{14+4}} = \frac{12}{\sqrt{18}} = \frac{4}{\sqrt{5}}$$

$$\Rightarrow \text{مساحت} = \frac{\pi}{\sqrt{5}} \quad S: \pi r^2 = \pi \frac{4}{5}$$

$$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{2}{-1} = -2 \rightarrow y - 3 = -2(x - 2) \rightarrow y = -2x + 7$$

$$m_{CH} = 1 \rightarrow y + 3 = x + 1 \rightarrow y = x - 2 \quad \text{نقطه } H: x - 2 = -2x + 7 \Rightarrow$$

$$\text{نقطه } H: \left(\frac{3+2}{2}, \frac{3+1}{2} \right) = H(2.5, 2.5)$$

$$\rightarrow x = \frac{17}{5}, y = \frac{3}{5}$$

$$\rightarrow H\left(\frac{17}{5}, \frac{3}{5}\right)$$

$$\text{طول } CH = \sqrt{\left(\frac{17}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{289}{25} + \frac{9}{25}} = \sqrt{\frac{298}{25}} = \frac{\sqrt{298}}{5}$$

$$\Rightarrow m_1 = \frac{-1}{m_2}$$

$$m_1 = \frac{m+1}{2m-4}$$

$$m_2 = \frac{2-3m}{2-m}$$

$$\rightarrow \frac{m+1}{2m-4} = \frac{2-m}{3m-2} \Rightarrow (m+1)(3m-2) = (2m-4)(2-m)$$

$$3m^2 + m - 2 = -2m^2 + 14m - 8 \Rightarrow 5m^2 - 13m + 6 = 0$$

$$\Rightarrow \Delta = 169 - 120 = 49 > 0 \Rightarrow \Delta \text{ حقیقی و جواب حقیقی ندارد.}$$

خطها را در به در با هم تقاطع می دهیم و مختصات رأس ها را در می یابیم $\Rightarrow A(1,1) \quad B(\omega, -1) \quad C(\frac{\omega}{\sqrt{3}}, \frac{\sqrt{3}}{\sqrt{3}})$ نقطه M: $(\frac{\omega + \frac{\omega}{\sqrt{3}}}{2}, \frac{-1 + \frac{\sqrt{3}}{\sqrt{3}}}{2}) \rightarrow M(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}})$ طول $\overline{AM} = \sqrt{(\frac{\sqrt{3}}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^2} = \frac{\sqrt{\omega^2 + 1}}{\sqrt{3}} = \frac{\omega + \sqrt{3}}{6}$ $m_{BC} = -1 \rightarrow m_{AH} = 1 \quad \overline{AH}: y = x$ نقطه H تقاطع BC و AH: نقطه H $\rightarrow x = y, y = x$ $\rightarrow H(2,2)$ طول $AH = \sqrt{1+1} = \sqrt{2} \Rightarrow \frac{AM}{AH} = \frac{\frac{\omega + \sqrt{3}}{6}}{\sqrt{2}} = \frac{\omega}{\sqrt{2}}$	
$y = -\frac{1}{\sqrt{3}}x + b \rightarrow y + \frac{1}{\sqrt{3}}x - b = 0$ $\Rightarrow y = -\frac{1}{\sqrt{3}}x - \omega$ حرف نسبتاً: $-\omega \rightarrow B(0, -\omega)$ طول نسبتاً: $-1 \rightarrow A(-1, 0)$ فاصله A و B: $\sqrt{(1-0)^2 + (\omega)^2} = \sqrt{1+\omega^2} = \omega\sqrt{\omega}$	$\frac{ ax+by+c }{\sqrt{a^2+b^2}} \rightarrow \frac{ -b }{\sqrt{1+(\frac{1}{\sqrt{3}})^2}} = \frac{ -b }{\frac{2}{\sqrt{3}}} = \sqrt{2}$ $\rightarrow -b = \omega \rightarrow b = \pm \omega$ چون فاصله ی خاصه میان ω و $-\omega$ تفاوتی ندارد.
$y = \frac{4}{m}x + 2$ $y = (1-m)x + 2$ فرض از مبدأ هر دو برابر است پس یکدیگر را در نقطه $(0,2)$ قطع می کنند $\Rightarrow$ ناممکنی طول از مبدأ فقط $\frac{\omega}{\sqrt{3}}$ است $\Rightarrow y = \frac{4}{m}x + 2, x=0, y=0 \Rightarrow x_1 = -\frac{m}{4}$ $y = (1-m)x + 2, y=0 \Rightarrow x_2 = -\frac{2}{1-m}$ $\Rightarrow \frac{m^2 - m + 4}{ 1-m } = \omega$ $m^2 + 4m - 1 = 0 \Rightarrow m = \frac{-4 \pm \sqrt{16+4}}{2} = -2 \pm \sqrt{5}$ $m^2 - 4m + 9 = 0 \Rightarrow m = 3$	$\Rightarrow y_x = \frac{\omega}{\sqrt{3}} \Rightarrow \frac{\omega}{\sqrt{3}} = \frac{\omega}{\sqrt{3}} \Rightarrow \frac{\omega}{\sqrt{3}} = \frac{\omega}{\sqrt{3}}$ $\Rightarrow \frac{ \frac{-2}{1-m} - \frac{-m}{4} }{ \frac{1}{1-m} - \frac{1}{4} } = \frac{ \frac{1-(m^2-m+4)}{12(1-m)} }{\frac{m^2-m+4}{12 1-m }} = \frac{m^2-m+4}{2 1-m } = \frac{\omega}{\sqrt{3}}$ $\Rightarrow (-2+\sqrt{5})(-2-\sqrt{5})(3) = -30$
$P: \frac{y+2}{r} \rightarrow Q: \frac{y}{r}$ $P: \frac{-1+y}{r} \rightarrow Q: \frac{y}{r} \Rightarrow B'(3, -3)$ $y' \parallel y \rightarrow m_{y'} = m_y \Rightarrow m_{y'} = 2 \Rightarrow y' + 3 = 2(x-3) \Rightarrow y' = 2x - 9$	نقطه $A(1, -1)$ روی خط y' است. آن را نسبت به M قرینه می کنیم.
$m_{AH} = -1 \rightarrow y - 2 = -(x - 1) \Rightarrow y = -x + 3$ A', D نقطه H وسط $A'D$ است $\Rightarrow -1 = \frac{1+A'}{2} \Rightarrow A' = -3$ $P: \frac{y+2}{r} \rightarrow Q: \frac{y}{r} \Rightarrow y_{A'} = 4$ $\rightarrow A'(-3, 4) \Rightarrow a = -3, b = 4 \Rightarrow 2b - a = 2(4) - (-3) = 11$	نقطه H: $-x + 3 = x + \omega \Rightarrow x = -1$ $y = 4$ $\Rightarrow H(-1, 4)$