

19, 25

$$2. (a-1)x^2 + x + 3$$

$$x_1, x_2 = \frac{-b}{2a} = \frac{-1}{2a-2}, 2 \rightarrow 4a-4 \rightarrow -1 \rightarrow \left(a = \frac{3}{4}\right) \checkmark$$

(2)

1

$$y_1 = \frac{1}{4}x^2 + x + 3 \rightarrow x_1 + 6, -2 \xrightarrow{\text{طول}} (x_1, 6) \checkmark$$

چون درجه‌های عبارت‌ها 2 و 1 است پس $f(x)$ حتماً $mx^2 - 5x + m$ دارد. $m < 0$ و $a < 0$ و $\Delta > 0$ و موارد درجه‌ها را در نظر بگیریم

(2)

2

$$m < 0, 25 - 4m^2 > 0 \rightarrow m < 0, \frac{-5}{2} < m < \frac{5}{2} \xrightarrow{\text{اشتراک}} \boxed{-\frac{5}{2} < m < 0} \checkmark$$

$$S = \alpha + \beta = \frac{3-\sqrt{5}}{3} + \frac{3+\sqrt{5}}{3} = \frac{6}{3} = 2$$

$$P = \alpha \cdot \beta = \left(\frac{3-\sqrt{5}}{3}\right) \cdot \left(\frac{3+\sqrt{5}}{3}\right) = \frac{9-5}{9} = \frac{4}{9} \rightarrow y = x^2 - 2x + \frac{4}{9}$$

(2)

3

$$\boxed{y = 9x^2 - 18x + 4} \checkmark$$

$$2x^2 - 8x + m + 2 = 0$$

$$\alpha + \beta = 4$$

$$\checkmark \boxed{m = 4}$$

$$\alpha = 1, \beta = 3$$

$$\alpha = \frac{1}{4}m$$

$$8\alpha = 2m$$

$$\alpha \cdot \beta = \frac{m}{2} + 1 \rightarrow 3 \cdot \frac{m}{2} + 1 = \frac{m}{2} + 2$$

$$8\alpha - m - 2 = m - 2$$

$$2\alpha^2 + \alpha^2 + \beta^2 = 12 \rightarrow 2\alpha^2 + 16 - m - 2 = 12 \rightarrow 2\alpha^2 = m - 2$$

(2)

4

$$2. a(x-2)^2 + 9 \xrightarrow{(0,5)} 5 = 4a + 9 \rightarrow a = -1 \rightarrow y = -x^2 - 4x + 9$$

$$y = -x^2 - 4x + 9 \rightarrow \begin{array}{c} -1 \quad 5 \\ | \quad | \\ -6 \quad +6 \end{array} \rightarrow \boxed{x \in (-1, 5)} \checkmark$$

(2)

5

$$\begin{aligned}
 & \alpha x^2 - 7x + 2\beta = 0 \rightarrow \alpha + \beta = 7 \rightarrow 3 + \beta = 7 \rightarrow \beta = 4 \\
 & \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha \cdot \beta} = \frac{7}{\alpha \cdot \beta} \rightarrow \frac{7}{\alpha \cdot \beta} = \frac{7}{2} \rightarrow \alpha \cdot \beta = 2 \\
 & \alpha + \beta = 7, \alpha \cdot \beta = 2 \rightarrow \alpha = 6, \beta = 1 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 & x^2 + mx - 2m = 0 \rightarrow \alpha^2 + m\alpha - 2m = 0 \\
 & \alpha + \beta = m, \alpha \cdot \beta = -2m \\
 & \alpha^2 - m\beta = 8 \rightarrow \alpha^2 + \beta^2 = 8 + 2m \\
 & m^2 + 2m - 8 = 0 \rightarrow m = 2, -4 \rightarrow \alpha + \beta = m = 2 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 & f(x) = mx^2 + 4x + \frac{m}{2} + 7 \\
 & \frac{-\Delta}{4a} = 8 \rightarrow \frac{-(16 - 2m^2 - 28m)}{4m} = \frac{m^2 + 14m - 8}{2m} = 8 \rightarrow m = -2 \\
 & \checkmark K = 3 \rightarrow -2x^2 + 4x + 6 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 & y = a(x-2)^2 + 6 \quad (2, 4) \rightarrow 4 = 4a + 6 \rightarrow a = -\frac{1}{2} \\
 & y = -\frac{1}{2}x^2 + 2x + 6 \rightarrow y = -\frac{1}{2}x^2 + 2x + 4 \rightarrow S = 4, P = 8 \\
 & \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha \cdot \beta} = \frac{4}{-8} = -\frac{1}{2} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 & x^2 - (4a+4)x + (3a^2 + 6a + 3) = 0 \\
 & \downarrow x=2 \quad a = -\frac{1}{3} \rightarrow x^2 - 8x + 12 = 0 \rightarrow x = 2, 6 \quad \checkmark \\
 & 4 - 8a - 8 + 3a^2 + 6a + 3 = 0 \rightarrow 3a^2 - 2a - 1 = 0 \rightarrow a = 1, -\frac{1}{3}
 \end{aligned}$$