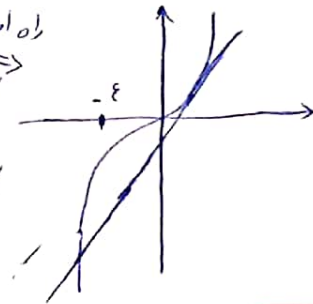


$$f(x) = \begin{cases} x^3 - 4 & ; x > a \\ 12x - 20 & ; x \leq a \end{cases}$$

برای برقراری عبارت
 $x^3 - 4 = 12x - 20 \Rightarrow x^3 - 12x + 16 = 0 \Rightarrow x = -4, 2$

با توجه به اعداد
 $a = [-4, +\infty)$



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 $x^3 - 4 = 12x - 20 \Rightarrow x^3 - 12x + 16 = 0 \Rightarrow x = -4, 2$

با توجه به اعداد
 $a = [-4, +\infty)$

$$f(x) = 3x + k \quad f^{-1}(2) = 4 \Rightarrow \frac{2-k}{3} = 4 \Rightarrow 2-k = 12 \Rightarrow k = -10 \Rightarrow f(x) = 3x - 10$$

$$f^{-1}(x) \Rightarrow y = 3x + k \Rightarrow x = \frac{y-k}{3} \Rightarrow 3x = y - k \Rightarrow y = \frac{3x+k}{1}$$

$$f(7) = 3(7) - 10 = 21 - 10 = 11$$

$$f(x) = 3x - 10 \Rightarrow f(f(x)) = f(3x - 10) = 3(3x - 10) - 10 = 9x - 30 - 10 = 9x - 40$$

$$\Rightarrow f(f(x)) = 9x - 40$$

$$f(x) = \frac{ax}{x-1} \quad f(2a)^{-1} = a \Rightarrow \frac{2a}{2a-a} = a \Rightarrow a = \frac{2a}{2a-a} = \frac{2a}{a} \Rightarrow a = 2$$

$$f(x) \Rightarrow y = \frac{ax}{x-1} \Rightarrow x = \frac{ay}{y-1} \Rightarrow xy - x = ay \Rightarrow xy - ay = x \Rightarrow y(x-a) = x$$

$$g = \{(3, 2), (1, 4), (9, 6)\} \rightarrow g^{-1} = \{(2, 3), (4, 1), (6, 9)\}$$

$$f = \{(3, 5), (4, 7), (6, 9), (9, 6)\} \rightarrow f^{-1} = \{(5, 3), (7, 4), (9, 6), (6, 9)\}$$

$$f \circ f^{-1} = \{(5, 5), (7, 7), (9, 9), (6, 6)\} \quad g \circ g^{-1} = \{(2, 2), (4, 4), (6, 6)\}$$

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$$f \circ f^{-1} = \{(5, 5), (7, 7), (9, 9), (6, 6)\} \quad g \circ g^{-1} = \{(2, 2), (4, 4), (6, 6)\}$$

$$f = \{(2, 4), (4, 8), (6, 10)\} \quad g = \{(2, 1), (4, 3), (6, 5)\} \rightarrow g^{-1} = \{(1, 2), (3, 4), (5, 6)\}$$

$$h = \{(1, 2), (3, 4), (5, 6), (7, 8)\}$$

$$f \circ g^{-1} = \{(1, 4), (3, 8), (5, 10)\}$$

$$1 \rightarrow 2 \rightarrow 4$$

$$3 \rightarrow 4 \rightarrow 8$$

$$5 \rightarrow 6 \rightarrow 10$$

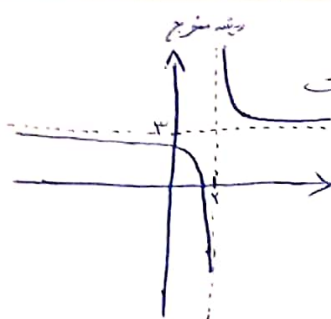
$$\frac{h}{f \circ g^{-1}} \xrightarrow{x=1} \frac{2}{4} = \frac{1}{2}$$

$$\frac{h}{f \circ g^{-1}} \xrightarrow{x=1} \frac{2}{4} = \frac{1}{2}$$

$$y = \frac{2x+1}{x-2}$$

$$\Downarrow x=2$$

$$\frac{4+1}{1} = 5$$

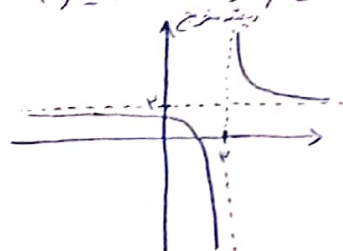


تابع یک به یک
دارد وارون پذیر است

$$y = \frac{2x+1}{x-2} \xrightarrow{\text{وارون}} x = \frac{2y+1}{y-2}$$

$$\Rightarrow xy - 2x = 2y + 1 \Rightarrow xy - 2y = 2x + 1$$

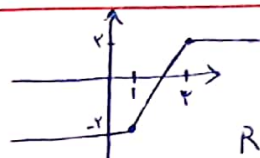
$$\Rightarrow y(x-2) = 2x+1 \Rightarrow y = \frac{2x+1}{x-2}$$



$$\Downarrow x=2$$

$$\frac{4+1}{-1} = -5$$

$$f(x) = |x-1| - |x-3| \Rightarrow \begin{cases} x-1-x+3=2 & x>3 \\ x-1+x-3=2x-4 & 1 \leq x < 3 \\ -x+1-x+3=-2 & x<1 \end{cases}$$



بارون پذیر [a,b] در بازه

$\Rightarrow$ بارون پذیر [1,3] در بازه

$$R_f = [-2, 2] \quad D_f = [1, 3]$$

$$f(x) = 2x-4 \quad 1 \leq x \leq 3 \xrightarrow{\text{وارون}} y = 2x-4 \Rightarrow x = \frac{y+4}{2} \Rightarrow y = \frac{x+4}{2}$$

$$\Rightarrow f(x)^{-1} = \frac{x+4}{2} \quad -2 \leq x \leq 2 \quad R_{f^{-1}} = [1, 3]$$

$$f(x) = \begin{cases} x^3+4 & x>1 \\ \varepsilon x-1 & x \leq 0 \end{cases}$$

رسم می کنیم



$$y = x^3+4 \xrightarrow{\text{مقلوب}} x = y^3+4$$

$$\Rightarrow y^3 = x-4 \Rightarrow y = \sqrt[3]{x-4}$$

$$y = \varepsilon x-1 \xrightarrow{\text{مقلوب}} x = \frac{y+1}{\varepsilon} \Rightarrow \varepsilon y = x+1$$

$$\Rightarrow y = \frac{x+1}{\varepsilon}$$

$$f(x)^{-1} = \begin{cases} \sqrt[3]{x-4} & x \geq 4 \\ \frac{x+1}{\varepsilon} & x \leq -1 \end{cases}$$

$$f(x) = x^2 - \frac{(x+1)^3}{x+3} \Rightarrow y = x^2 - \frac{(x+1)^3}{x+3} = \frac{-(x^3+3x^2+3x+1)}{x+3} + x^2 = \frac{x^3+3x^2-3x-1}{x+3}$$

$$\Rightarrow y = \frac{-(x^3+1)}{x+3} \xrightarrow{\text{وارون}} x = \frac{-y-1}{y+3} \Rightarrow xy + 3x = -y-1 \Rightarrow xy + 3y = -x-1$$

$$\Rightarrow y(x+3) = -x-1 \Rightarrow y = \frac{-x-1}{x+3}$$

$$f^{-1}(x) = \frac{ax+b}{cx+d} = \frac{-x-1}{x+3} = \frac{-1x-1}{1x+3}$$

$$f^{-1}(x) = \frac{-x-1}{x+3} \Rightarrow f(-2) = \frac{1-1}{-2+3} = \frac{0}{1} = 0 \Rightarrow b = -1 \quad (b < 0) \checkmark$$

$$f(x) = \frac{x}{x^2+1}$$

$$\frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1} \Rightarrow x_1x_2^2+x_1 = x_1^2x_2+x_2$$

$$\Rightarrow x_1x_2^2-x_1^2x_2+x_1-x_2 = 0 \Rightarrow x_1x_2(x_2-x_1) - (x_2^2-x_1^2) = 0 \Rightarrow (x_1x_2-1)(x_2-x_1) = 0$$

$$\Rightarrow x_1x_2 = 0 \Rightarrow x_1 = \frac{1}{x_2} \quad x_2 = 1 \Rightarrow f\left(\frac{1}{x_2}\right) = \frac{\frac{1}{x_2}}{\frac{1}{x_2^2}+1} = \frac{x_2}{1+x_2^2} \quad f(1) = \frac{1}{2} \Rightarrow$$

$$\text{ولی اگر بخواهیم وارون کنیم} \quad y = \frac{x}{x^2+1} \xrightarrow{\text{وارون}} x = \frac{y}{y^2+1} \Rightarrow xy^2+x = y \Rightarrow xy^2-y+x = 0$$

$$\Rightarrow y = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow y = \frac{1 \pm \sqrt{(-1)^2-4x^2}}{2x} \Rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x} \Rightarrow f(x)^{-1} = \frac{1 \pm \sqrt{1-4x^2}}{2x}$$