

الف) $n^2 - n \leq 2 \Rightarrow n^2 - n - 2 \leq 0 \Rightarrow (n-2)(n+1) \leq 0 \Rightarrow \boxed{n \leq 2}$ و $n \geq -1$
 $m \leq 3$

ب) $m \leq 2 \Rightarrow m-1 \leq 1 \Rightarrow a \leq -1$ $-1+2 \leq 1 \Rightarrow \boxed{n \leq 2}$

$\min n-1 \leq 2 \Rightarrow n+a \leq 2 \xrightarrow{n \leq 1} \boxed{a \leq 1}$

$\min n-1 \leq 2 \Rightarrow a n + a - 1 \leq 2 \xrightarrow{n \leq 0} a - 1 \leq 2$
 $\Rightarrow \boxed{a \leq 3}$

الف) $\xrightarrow{\text{تابع یکتا}} \text{یک به یک است} \xrightarrow{\text{عکس}} x = \frac{y+1}{y-2} \rightarrow y = \frac{x+1}{x-2}$

ب) $y \leq 2$ یک به یک نیست

سوال ۵

الف) $\begin{cases} y = x_1^3 + 2 \\ y = x_2^3 + 2 \end{cases} \Rightarrow x_1^3 + 2 = x_2^3 + 2 \rightarrow x_1 = x_2$ تابع یک به یک است
 $y-2 = x^3 \rightarrow x = \sqrt[3]{y-2} \rightarrow y = \sqrt[3]{x-2}$

ب) $\xrightarrow{\text{عکس}} \text{یک به یک نیست}$

سوال ۴

الف) $y = \sqrt{x-2}$ ————— تابع یک به یک است $y^2 = x-2$
 $x = y^2 + 2 \rightarrow y = \sqrt{x-2}, x \geq 2$ $\leftarrow R = [2, +\infty)$

ب) $y = x^2 - 4x + 3$ $y = (x-2)^2 - 1 \rightarrow \sqrt{y+1} = x-2 \rightarrow x = \sqrt{y+1} + 2$
 تابع یک به یک است $\rightarrow y = \sqrt{x-1} + 2$

$y = x + \sqrt{x} \rightarrow y = (\sqrt{x} + \frac{1}{\sqrt{x}})^2 - \frac{1}{x} \rightarrow y + \frac{1}{x} = (\sqrt{x} + \frac{1}{\sqrt{x}})^2$
 $\sqrt{y + \frac{1}{x}} = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow \sqrt{x} = \sqrt{y + \frac{1}{x}} - \frac{1}{\sqrt{x}} \rightarrow x = y + \frac{1}{x} + \frac{1}{x} - \sqrt{y + \frac{1}{x}}$
 $\rightarrow y = x + \frac{1}{x} - \sqrt{x + \frac{1}{x}} \Rightarrow a=1, b=\frac{1}{x}, c=-\frac{1}{x}$
 $a + 2b + c = 1 + 1 - 1 = 1$

$\begin{cases} y = \frac{x_1}{\sqrt{x_1^2-2}} \\ y = \frac{x_2}{\sqrt{x_2^2-2}} \end{cases} \Rightarrow \frac{x_1}{\sqrt{x_1^2-2}} = \frac{x_2}{\sqrt{x_2^2-2}} \xrightarrow{\text{مربع و ضرب}} \frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2}$

$\Rightarrow x_1^2 x_2^2 - 2x_1^2 = x_1^2 x_2^2 - 2x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$
 $y = \frac{x}{\sqrt{x^2-2}} \rightarrow x^2 = \frac{y^2}{y^2-1} \rightarrow y^2 = \frac{2x^2}{x^2-1}$
 $y = \pm \sqrt{\frac{2x^2}{x^2-1}} \rightarrow y = \frac{\sqrt{2x^2}}{\sqrt{x^2-1}}$
 یک به یک است $\rightarrow y = \frac{\sqrt{2x^2}}{\sqrt{x^2-1}}$

$f^{-1}(x) = \frac{x}{4} \Rightarrow \frac{x}{1+|x|} = \frac{x}{4} \Rightarrow 4x = -2 + 2x \Rightarrow 2x = -2 \Rightarrow x = -\frac{1}{2}$
 $\Rightarrow f(-\frac{1}{2}) = \frac{-\frac{1}{2}}{4}$

$g(x) = \frac{x}{4} \Rightarrow \sqrt{x-1} = \frac{x}{4} \Rightarrow \sqrt{x} = \frac{x}{4} + 1 \Rightarrow \sqrt{x} = \frac{x+4}{4} \Rightarrow x = \frac{9}{4}$
 $f(\frac{x}{4}) + g^{-1}(\frac{x}{4}) = \frac{9}{4} - \frac{1}{4} = \frac{2x-4}{4} = \frac{2x}{4} = \frac{x}{2} \Rightarrow g^{-1}(\frac{x}{4}) = \frac{9}{4}$

$g(x) = 12 \rightarrow f(x) + \sqrt{f(x)} = 12 \rightarrow f(x) = 9$
 $f^{-1}(9) = \sqrt{9-1} = \sqrt{8} = 2$