

۱۹, ۵

نام و نام خانوادگی: شماره کلاس: پاسخنانه تشریحی تکلیف شماره:
 ۱۹, ۵

الف) $f = \{ (3, 2), (n, 2), (3, n-n), (n, 2), (-1, 1) \} \Rightarrow n^2 - n = 2 \Rightarrow n^2 - n - 2 = 0$

$(n-2)(n+1) \Rightarrow n=2$ و $n=-1$ ✓

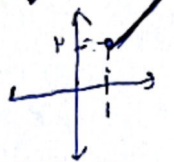
ب) $f = \{ (-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 3) \}$

$m=2$ ✓ $2-1=1 \Rightarrow (a, 1) \Rightarrow a=-1$ ✓

$-1+2=1 \Rightarrow (1, m) \Rightarrow m=2$ ✓

الف) $f(m) = \begin{cases} 3m-1 & m > 1 \\ a+2 & m \leq 1 \end{cases} \Rightarrow 3m-1 > 1 \Rightarrow R_f = [2, +\infty)$

ب) $a+2 > 1 \Rightarrow (-\infty, 1) \Rightarrow 3(1)-1 > 1+2 \Rightarrow a < 1$ ✓



$a > 0$

$a \neq 0$

تعیین $a+2-1$ بر مبنای a

$f(m) = \begin{cases} 3m-1 & m > 1 \\ a+2 & m \leq 1 \end{cases}$

$f(m_1) = f(m_2)$

$a+2-1 = 3m_2-1 \Rightarrow a(m_1+1) = 3m_2$

$a_1 \leq 0 \Rightarrow a+1 \leq 1 \Rightarrow a(m_1+1) \leq a \Rightarrow 3m_2 > 3 \Rightarrow a < 3$

$a > 0 \Rightarrow a < 3 \Rightarrow a \in (0, 3)$ ✓

الف) $y = a^2 + 2 \Rightarrow \begin{cases} a_1^2 + 2 = y \\ a_2^2 + 2 = y \end{cases} \Rightarrow a_1^2 + 2 = a_2^2 + 2 \Rightarrow a_1^2 = a_2^2 \Rightarrow a_1 = a_2$ ✓

$y = a^2 + 2 \Rightarrow y-2 = a^2 \Rightarrow a = \sqrt{y-2} \Rightarrow y = \sqrt{a-2}$ ✓

ب) $a^2 - 2a + 2 = y \Rightarrow$ تابع درجه دوم و معی مرتبه زوج است ✓

الف) $y = \frac{3a+1}{a-2} \Rightarrow y(a-2) = 3a+1$

$\Rightarrow ya - 2a = 3a + 1 \Rightarrow a(y-3) = 1+2y \Rightarrow a = \frac{1+2y}{y-3} \Rightarrow y = \frac{1+2a}{a-3}$ ✓

ب) $y = \frac{2+2a}{a+2} \Rightarrow \frac{2(1+a)}{a+2} = 2$ ✓

الف) $y = \sqrt{u-3} \Rightarrow u = \sqrt{y-3} \Rightarrow u^2 = y-3 \Rightarrow u^2 + 3 = y$ باید به شکل دریا که می بینید

ب) $y = u^2 - 2u + 3 \quad u \in [1; +\infty) \Rightarrow$ بررسی می کنیم تغییر کرده است

$y = u^2 - 2u + 3 + 1 - 1 \Rightarrow y = u^2 - 2u + 3 - 1 \Rightarrow y = (u-1)^2 - 1 \Rightarrow u-1 = \pm \sqrt{y+1}$

$u = 1 \pm \sqrt{y+1} \Rightarrow f^{-1}(u) = 1 + \sqrt{u+1}$ چون آر بی بی

$f(u) = u + \sqrt{u} \Rightarrow u \in (0; +\infty) \Rightarrow \sqrt{u} = y - u \Rightarrow u = (y-u)^2 \Rightarrow u = y^2 - 2uy + u^2$

$u^2 - 2uy - u + y^2 = 0 \Rightarrow u^2 - (2y+1)u + y^2 = 0 \Rightarrow u = \frac{2y+1 \pm \sqrt{(2y+1)^2 - 4y^2}}{2}$

$\Rightarrow \frac{2y+1 \pm \sqrt{4y+1}}{2} \Rightarrow f^{-1}(y) = \frac{2y+1 - \sqrt{4y+1}}{2} \Rightarrow f^{-1}(y) = y + \frac{1}{2} - \frac{1}{2}\sqrt{4y+1}$

$-\frac{1}{2}\sqrt{4y+1} = -\sqrt{y+\frac{1}{4}} \Rightarrow f^{-1}(u) = u + \frac{1}{2} - \sqrt{u+\frac{1}{4}} \Rightarrow a=1 \quad b=\frac{1}{2} \quad c=-\frac{1}{4}$
 $\Rightarrow 1 + 2(\frac{1}{2}) + 4(-\frac{1}{4}) = 1 + 1 - 1 = 1$

$g = \frac{u}{\sqrt{u^2-4}} \Rightarrow f(u_1) = f(u_2) \Rightarrow \frac{u_1}{\sqrt{u_1^2-4}} = \frac{u_2}{\sqrt{u_2^2-4}} \Rightarrow \frac{u_1^2}{u_1^2-4} = \frac{u_2^2}{u_2^2-4} \Rightarrow$

$u_1^2 u_2^2 - 4u_1^2 = u_1^2 u_2^2 - 4u_2^2 \Rightarrow -4u_1^2 = -4u_2^2 \Rightarrow u_1 = \pm u_2$ مختلف از هم نیستند

$g = \frac{u}{\sqrt{u^2-4}} \Rightarrow u = \frac{y}{\sqrt{y^2-4}} \Rightarrow u^2 = \frac{y^2}{y^2-4} \Rightarrow u^2 y^2 - 4u^2 = y^2$

$y^2 u^2 - y^2 = 4u^2 \Rightarrow y^2(u^2-1) = 4u^2 \Rightarrow y^2 = \frac{4u^2}{u^2-1} \Rightarrow y = \pm \sqrt{\frac{4u^2}{u^2-1}} \Rightarrow y = \frac{\sqrt{2}u}{\sqrt{u^2-1}}$

$g(u) = \sqrt{u-1} \quad f^{-1}(u) = \frac{u}{1+|u|} \Rightarrow f(-\frac{1}{2}) + g^{-1}(-\frac{1}{2})$

$\sqrt{u-1} = -\frac{1}{2} \Rightarrow \sqrt{u} = \frac{1}{2} \Rightarrow u = \frac{1}{4}$

$\frac{u}{1+|u|} = -\frac{1}{2} \Rightarrow -2 + 2|u| = u \Rightarrow -2 + 2u = u \Rightarrow -2 = -u \Rightarrow u = 2$

$\frac{1}{4} + (-\frac{1}{2}) \Rightarrow \frac{1u-20}{u2} = -\frac{1}{2}$

$f^{-1}(u) = \sqrt{u-1} \quad g(u) = f(u) + \sqrt{f(u)} \Rightarrow g^{-1}(12) = ?$ فقط فقط

$f(u) + \sqrt{f(u)} = 12 \Rightarrow \sqrt{f(u)} = t \Rightarrow t^2 + t - 12 = 0 \Rightarrow (t+4)(t-3) \Rightarrow t = -4 \quad t = 3$

$\sqrt{f(u)} = 3 \Rightarrow f(u) = 9 \Rightarrow f^{-1}(u) = \sqrt{u-1} \Rightarrow f^{-1}(9) = \sqrt{9-1} = 2 \Rightarrow f(2) = 9$

$g(2) = 12 \quad g^{-1}(12) = 2$