

یا در صورت

۲۰ عالی نوشتن

برای هر مقدار

الف: $f = \{(3, 2), (n, 0), (3, n^2 - n), (m, 2), (-1, 4)\} \Rightarrow m = 3$ ✓
 $n^2 - n = 2$
 $n^2 - n - 2 = 0$
 $(n - 2)(n + 1) = 0$
 $n = 2$ ✓
 $n = -1$ ✓
 $m = 3$ ✓
 $n = 2$ ✓
 $n = -1$ ✓

ب) $f = \{(-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 3)\}$

$f(a) = \begin{cases} 2a-1 & ; a \geq 1 \\ a^2+a-1 & ; a < 1 \end{cases}$

$(-\infty, 1) \Rightarrow a = 1 \Rightarrow a = (-\infty, 1]$ ✓

$f(a) = \begin{cases} 2a-1 & ; a \geq 1 \\ a^2+a-1 & ; a < 1 \end{cases}$

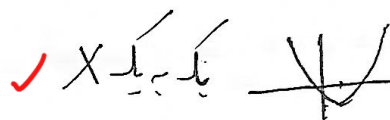
$[1, +\infty)$

$(-\infty, 1)$

$(0, 3)$

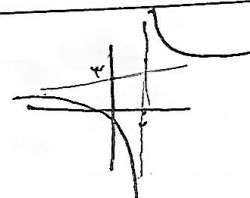
$y = x^3 + 2 \Rightarrow x = y^{\frac{1}{3}} + 2 \Rightarrow y^{\frac{1}{3}} = x - 2 \Rightarrow y = (x - 2)^3$ ✓

$y = x^3 - (x+2)^3$
 $x = -2$

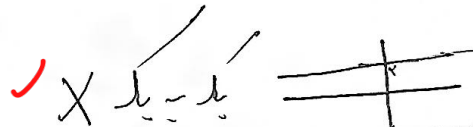


$y = \frac{x+1}{x-2} \Rightarrow x = \frac{y+1}{y-2} \Rightarrow y = -\frac{-y-1}{2-y} = \frac{y+1}{2-y}$ ✓

$y = \frac{x+1}{x-2}$



$y = \frac{x+2}{x+2} \Rightarrow \frac{y+2}{y+2}$

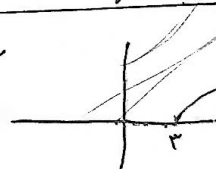


$y = \sqrt{x-3}$

$\sqrt{x-3} = \sqrt{x-3}$
 $|x-3| = |x-3|$
 $|x-3|^2 = |x-3|^2$

$|a| = |a|$
 $a = \pm a \Rightarrow a = a$
 $-a = -a$

$y = \sqrt{x-3}$



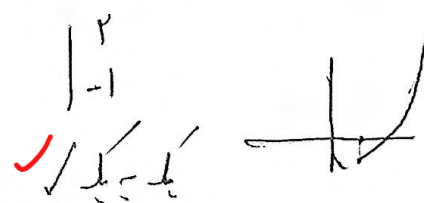
$x = \sqrt{y-3} \Rightarrow x^2 = |y-3| \Rightarrow x^2 + 3 = |y| \Rightarrow y = x^2 + 3 ; x \geq 0$ ✓

$y = x^2 - (x+2)^2$ $x \geq 2$

$x = y^2 - (y+2)^2$
 $x = (y-1)^2 - 1$

$(y-1)^2 = x+1$
 $y-1 = \sqrt{x+1}$

$y = \sqrt{x+1} + 1$ ✓



$$f(x) = a + \sqrt{x}$$

2).

$$f^{-1}(a) = ax + b = \sqrt{a-6}$$

$$x = y + \sqrt{y}$$

$$\left(y + \frac{1}{y}\right)^2 = \frac{1}{f} = x$$

$$\sqrt{y} + \frac{1}{\sqrt{y}} = \sqrt{x + \frac{1}{f}}$$

$$\sqrt{y} = \sqrt{x + \frac{1}{f}} - \frac{1}{\sqrt{y}} \Rightarrow y = x + \frac{1}{f} + \frac{1}{f} - \frac{1}{\sqrt{x + \frac{1}{f}}} \cdot \frac{1}{\sqrt{y}}$$

$$a = 1$$

$$b = \frac{1}{f}$$

$$c = -\frac{1}{f}$$

$$|x| = 1 \Rightarrow (1)$$

$$y = \frac{x}{x^2 - 2}$$

$$y_1 = \frac{x_1}{x_1^2 - 2}$$

$$y_2 = \frac{x_2}{x_2^2 - 2}$$

$$\frac{x_1}{x_1^2 - 2} = \frac{x_2}{x_2^2 - 2}$$

$$\frac{x_1^2}{x_1^2 - 2} = \frac{x_2^2}{x_2^2 - 2}$$

$$\frac{x_1^2}{x_1^2 - 2} = \frac{x_2^2}{x_2^2 - 2} \Rightarrow x_1^2 = x_2^2$$

✓ $x_1 = \pm x_2$: حالتی که در صورت و مخرج برابر می شود و قاعده است پس باید یک یک کرد ✓

$$x = \frac{y}{y^2 - 2} \Rightarrow x^2 = \frac{y^2}{y^2 - 2} \Rightarrow y^2 = -\frac{-2x^2}{x^2 - 1} \Rightarrow y^2 = \frac{2x^2}{x^2 - 1} \Rightarrow y = \pm \frac{\sqrt{2}x}{\sqrt{x^2 - 1}}$$

$$f^{-1}(x) = \frac{x}{1+|x|}$$

$$g(x) = \sqrt{x} - 1$$

$$f\left(-\frac{x}{a}\right) + g^{-1}\left(-\frac{x}{a}\right)$$

$$-\frac{x}{a} + \frac{a}{x} = \frac{-x^2 + a^2}{ax} = -\frac{x^2 - a^2}{ax}$$

$$\frac{x}{1+|x|} = -\frac{x}{a} \Rightarrow -ax = x(1+|x|)$$

$$-x^2 = x \Rightarrow -x^2 = x \Rightarrow x = -\frac{x}{a} \Rightarrow x = -\frac{a}{x} \Rightarrow x^2 = -a$$

$$\sqrt{x} - 1 = -\frac{x}{a} \Rightarrow \sqrt{x} = -\frac{x}{a} + 1 \Rightarrow x = \frac{a}{x}$$

$$f^{-1}(x) = \sqrt{x-1}$$

$$g(x) = f(x) + \sqrt{f(x)}$$

$$g^{-1}(12) = (2)$$

$$f(x) + \sqrt{f(x)} = 12$$

$$\sqrt{f(x)} = t$$

$$t + \sqrt{t} = 12$$

$$(t + \sqrt{t})(t - \sqrt{t}) = 0$$

$$t = -\sqrt{t} \Rightarrow t = 0$$

$$f(x) = -\sqrt{t} \Rightarrow f(x) = 0 \Rightarrow f(x) = 9$$

$$f(x) = 9$$