

9 الف) $f\left(\frac{r+1}{n-r}\right) = t \Rightarrow \frac{r+1}{n-r} = t \Rightarrow \frac{rt+1}{t-r} = n$
 $f(t) = t\left(\frac{rt+1}{t-r}\right) + \Delta \Rightarrow f(t) = \frac{rt+1}{t-r} + \frac{\Delta t - 1}{t-r} = \frac{rt-1}{t-r}$
 $f(x) = \frac{rt-1}{x-r}$

ب) $f\left(n + \frac{1}{n}\right) = n^r + \frac{1}{n^r} \Rightarrow f\left(n + \frac{1}{n}\right) = \left(n + \frac{1}{n}\right)^r - r\left(n + \frac{1}{n}\right)$
 $\left(n + \frac{1}{n}\right)^r = n^r + \frac{1}{n^r} + r\left(n + \frac{1}{n}\right) \Rightarrow n^r + \frac{1}{n^r} = \left(n + \frac{1}{n}\right)^r - r\left(n + \frac{1}{n}\right)$
 $\Rightarrow f(n) = n^r - rn$

10
 $g \circ f(n) = 0 \Rightarrow g(f(n)) = 0 \Rightarrow f(n) = 2\sqrt{2} \Rightarrow n\sqrt{n} = 2\sqrt{2} \Rightarrow n = 2$
 $f(n) = 1 \Rightarrow n\sqrt{n} = 1 \Rightarrow n = 1$
 $\Rightarrow$ $1, 2 =$ x های که نمودار g را قطع می کنند $\Rightarrow (2-1=1)$

19, 5

تکلیف سبکی 10. اسیریلوین بنام بزرگم بیان B

7 الف) $\{(2, 2), (n, 5), (3, n^2-n), (m, 2), (-1, 4)\}$

$n^2-n=2 \Rightarrow n=2$ ✓, $m=3$ ✓

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ب) $\{(-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 2)\}$

$m=2 \Rightarrow m-1=1 \Rightarrow a=-1 \Rightarrow a+2=1 \Rightarrow n=2$ ✓

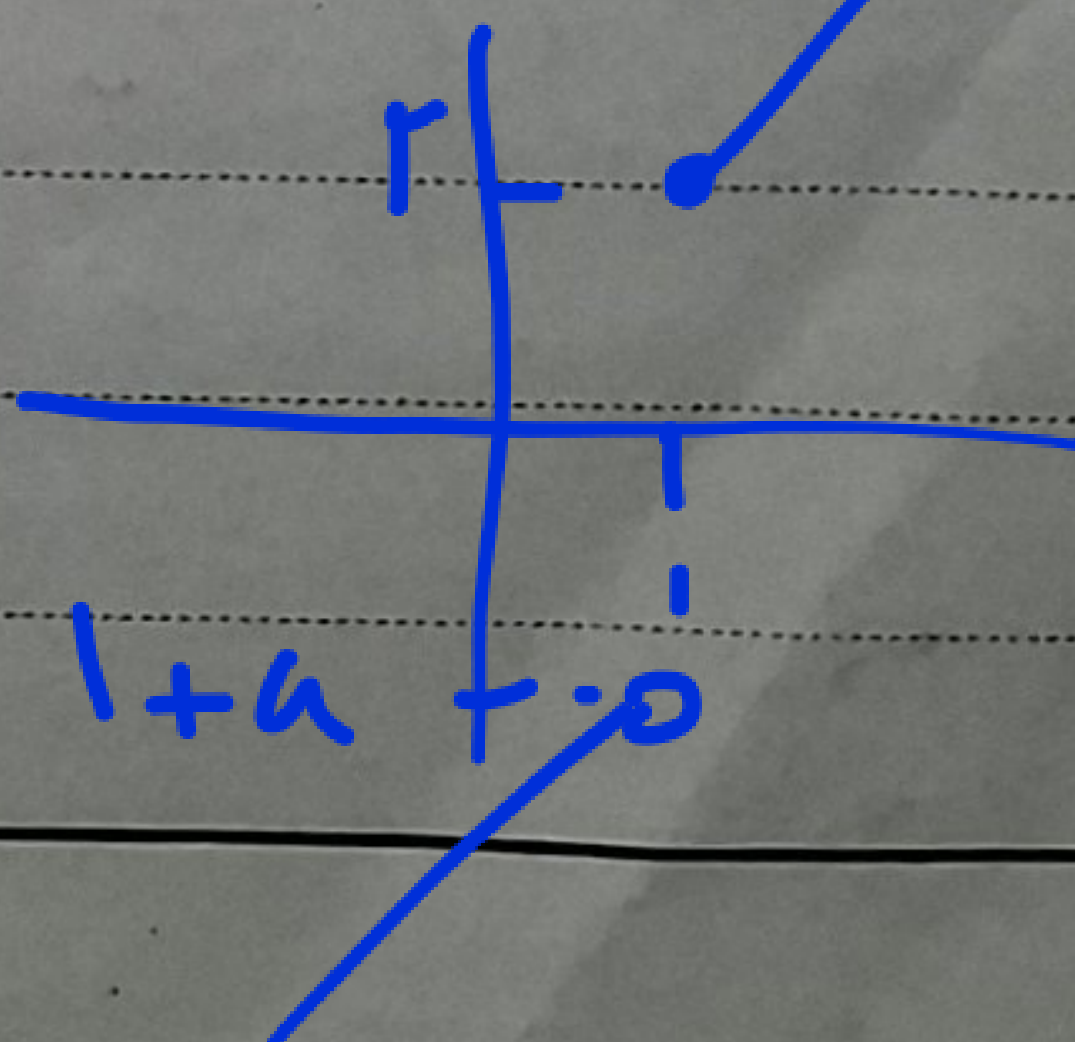
2 $f(x) = \begin{cases} rx-1 & ; x \geq 1 \\ x+a & ; x < 1 \end{cases}$, a چقدر؟

1, 1/2

$rx-1 \xrightarrow{x=1} = 2 \Rightarrow x+a \xrightarrow{x=1} < 2$

$\Rightarrow a < 1 \Rightarrow (-\infty, 1)$

صودی هم اوله!

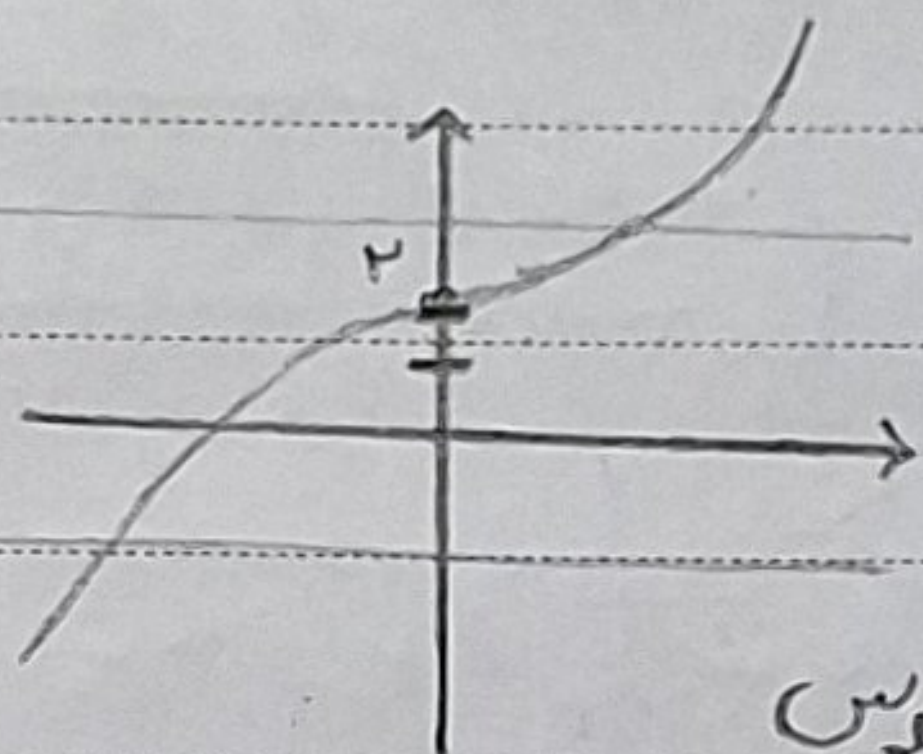


$$f(x) = \begin{cases} x^2 - 1 & x \geq 1 \\ ax + a - 1 & x \leq 0 \end{cases} \quad a = ? \quad f(x) = \begin{cases} x^2 - 1 & x \geq 1 \Rightarrow [1, +\infty) \\ ax + a - 1 & x \leq 0 \Rightarrow (-\infty, a-1] \end{cases}$$

$$a - 1 < 1 \Rightarrow a < 2 \quad \text{نقطه}$$

$$① \cap ② : \boxed{0 < a < 2} \quad \checkmark$$

$$④ \quad y = x^2 + 2 \quad \text{الف}$$



همه خط موازی با محور x آنرا قطع نمی کند یک به یک است
 $y = \sqrt{x-2}$ خط عکس

تابع $y = x^2 - 4x + 2$ است در نتیجه خطوط موازی با محور x آنرا قطع می کنند پس نتیجه می گیریم تابع یک به یک نیست

$$y = x^2 - 4x + 2 \Rightarrow y = (x-2)^2 - 2$$

$$\begin{cases} y = (x_1 - 2)^2 - 2 \\ y = (x_2 - 2)^2 - 2 \end{cases} \Rightarrow (x_1 - 2)^2 - 2 = (x_2 - 2)^2 - 2 \Rightarrow x_1 - 2 = \pm (x_2 - 2)$$

$$⑤ \quad y = \frac{x^2 + 1}{x - 2}$$

$$x = \frac{y - 1}{x + 2} = \frac{-y + 1}{x + 2} \Rightarrow y^{-1} = \frac{1 + 2x}{x - 2}$$

تابع $y = \frac{x^2 + 1}{x - 2}$ یک به یک نیست

$$\text{ب) } y = \frac{x^2 + 2x}{x + 2} \Rightarrow y = \frac{x(x + 2)}{x + 2} \Rightarrow y = x \Rightarrow \text{یک به یک نیست}$$

$$⑥ \quad y = \sqrt{x-3} \Rightarrow y = \sqrt{x_1-3}, y = \sqrt{x_2-3} \Rightarrow x_1 - 3 = x_2 - 3 \Rightarrow x_1 = x_2$$

$\Rightarrow$ یک به یک است

$$\Rightarrow y^{-1} = x^2 + 3 \quad \checkmark$$

$$\text{ب) } y = x^2 - 4x + 3 \quad ; \quad x \in [2, +\infty)$$

تابع در دامنه داده شده یک به یک است

$$\Rightarrow y = x^2 - 4x + 3 \Rightarrow y = (x-2)^2 - 1 \Rightarrow x - 2 = \pm \sqrt{y+1}$$

$$\Rightarrow x = 2 + \sqrt{y+1} \Rightarrow y^{-1} = 2 + \sqrt{y+1} \quad \checkmark$$

دامنه مثبت است
 پس یکتا

⑦ $f(x) = x + \sqrt{x}$, $f^{-1}(x) = ax + b - \sqrt{x-c}$; $x \in (0, +\infty)$ (2)
 $a + rb + c = ?$

$$f(x) = x + \sqrt{x} \xrightarrow{t = \sqrt{x}} y = t^2 + t \Rightarrow t^2 + t - y = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1+4y}}{2}$$

$$\Rightarrow t = \frac{-1 + \sqrt{1+4y}}{2} \Rightarrow x = t^2 = \left(\frac{-1 + \sqrt{1+4y}}{2} \right)^2$$

$$f^{-1}(x) = \frac{1+2x - \sqrt{1+4x}}{2} = x + \frac{1}{2} - \frac{1}{2}\sqrt{1+4x} = x + \frac{1}{2} - \sqrt{x + \frac{1}{4}}$$

$$\Rightarrow a = 1, b = \frac{1}{2}, c = -\frac{1}{4} \Rightarrow a + rb + c = 1 + 1 - 1 = 1 \quad \checkmark$$

⑧ $y = \frac{x}{\sqrt{x^2 - r}} \Rightarrow \begin{cases} y = \frac{x_1}{\sqrt{x_1^2 - r}} \\ y = \frac{x_2}{\sqrt{x_2^2 - r}} \end{cases} \Rightarrow \frac{x_1}{\sqrt{x_1^2 - r}} = \frac{x_2}{\sqrt{x_2^2 - r}} \Rightarrow \frac{x_1^2}{x_1^2 - r} = \frac{x_2^2}{x_2^2 - r} \Rightarrow \frac{x_1^2}{x_1^2 - r} = \frac{x_2^2}{x_2^2 - r}$ (1, 2, 3)

$$\Rightarrow x_1^2 (x_2^2 - r) = x_2^2 (x_1^2 - r) \Rightarrow x_1^2 x_2^2 - r x_1^2 = x_2^2 x_1^2 - r x_2^2 \Rightarrow x_1^2 x_2^2 - r x_1^2 = x_1^2 x_2^2 - r x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$$

✓. است. ←

$$y^{-1}: x = \frac{y}{\sqrt{y^2 - r}} \Rightarrow x \sqrt{y^2 - r} = y \Rightarrow x^2 (y^2 - r) = y^2 \Rightarrow x^2 y^2 - r x^2 = y^2$$

$$\Rightarrow y^2 - x^2 y^2 = r x^2 \Rightarrow y^2 = \frac{r x^2}{1 - x^2} \Rightarrow y = \pm \sqrt{\frac{r x^2}{1 - x^2}}$$

$$\Rightarrow y^{-1} = \pm \sqrt{\frac{r x^2}{1 - x^2}} \quad \star \text{ مثل شرط بالا در حد } \checkmark$$

⑨ $f^{-1}(x) = \frac{x}{1+|x|}$, $g(x) = \sqrt{x} - 1 \Rightarrow f(x): y = \frac{x}{1+|x|}$ (2)

$$\Rightarrow x = \frac{y}{1+|y|} \xrightarrow{x = \frac{r}{\Delta}} \frac{-r}{\Delta} = \frac{y}{1+|y|} \Rightarrow -r - r|y| = \Delta y \Rightarrow \Delta y + r|y| + r = 0$$

$$\Rightarrow \begin{cases} \Delta y + r = 0 \Rightarrow y = -\frac{r}{\Delta} & y > 0 \\ r y + r = 0 \Rightarrow y = -\frac{r}{r} & y < 0 \end{cases} \Rightarrow f\left(-\frac{r}{\Delta}\right) = -\frac{r}{r} \quad \checkmark$$

$$g^{-1}(x) \Rightarrow y = \sqrt{x} - 1 \xrightarrow{x = \frac{r}{\Delta}} \sqrt{y} - 1 = \frac{-r}{\Delta} \Rightarrow \frac{r}{\Delta} = \sqrt{y} \Rightarrow y = \frac{r^2}{\Delta^2} = g^{-1}\left(-\frac{r}{\Delta}\right)$$

$$f\left(-\frac{r}{\Delta}\right) + g^{-1}\left(-\frac{r}{\Delta}\right) = \frac{-\Delta + r}{\Delta} = \frac{-r}{\Delta} \quad \checkmark$$

Subject.

Date.

$$(10) \quad f^{-1}(x) = \sqrt[r]{x-1}, \quad g(x) = f(x) + \sqrt{f(x)}$$

(1)

$$f(x) \rightarrow y, \quad \sqrt[r]{x-1} \Rightarrow x-1 = y^r \Rightarrow x = y^r + 1 \Rightarrow f(x) = x^r + 1$$

$$g(x) = x^r + 1 + \sqrt{x^r + 1} \xrightarrow{a=x, y=1r} g(a) = a^r + 1 + \sqrt{a^r + 1} = 1r$$

$$\Rightarrow a^r + \sqrt{a^r + 1} = 1 \Rightarrow a = r \Rightarrow g(r) = 1r \Rightarrow g^{-1}(1r) = r \quad \checkmark$$