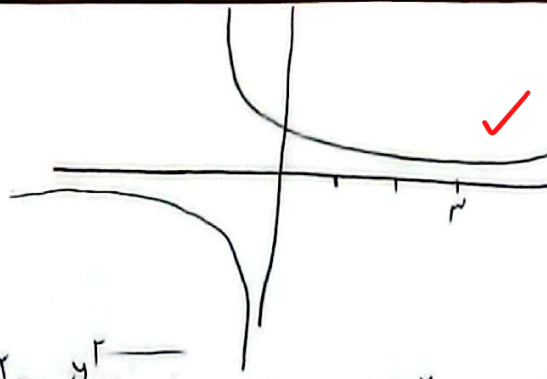


$$y = \frac{x}{\sqrt{x^2 - 1}}$$



$$x \sim \frac{1}{x}$$

(2)

$$x = \frac{y}{\sqrt{y^2 - 1}}$$

$$x^2 = \frac{y^2}{y^2 - 1}$$

$$y^2 = \frac{a^2 + r x^2}{x^2 - 1} \rightarrow y = \frac{x \sqrt{r}}{\sqrt{x^2 - 1}}$$

$$f^{-1}(x) = \frac{x}{1 + |x|}$$

$$g(x) = \sqrt{x} - 1 - \frac{r}{r} f\left(\frac{r}{\sqrt{x}}\right) + g^{-1}\left(\frac{r}{\sqrt{x}}\right) \frac{r}{r\sqrt{x}}$$

$$-\frac{r}{\sqrt{x}} = \sqrt{x} - 1 \quad \sqrt{x} = \frac{r}{\sqrt{x}} \quad \boxed{x = \frac{r}{r\sqrt{x}}}$$

$$-\frac{r}{\sqrt{x}} = \frac{x}{1 + |x|}$$

$$2x = -r - r|x|$$

$$2x + r|x| = -r \rightarrow x = -\frac{r}{r}$$

$$-\frac{r}{r} + \frac{r}{r\sqrt{x}} = \frac{-20 + 20}{\sqrt{x}} = -\frac{r}{\sqrt{x}}$$

$$f^{-1}(x) = \sqrt{x} - 1$$

$$g(x) = f(x) + \sqrt{f(x)}$$

$$\boxed{x^2 + 1 = +}$$

$$x = \sqrt{y - 1}$$

$$f(x) = x^2 + 1$$

$$x^2 + 1 + \sqrt{x^2 + 1} = 1r$$

$$\rightarrow x = \sqrt{\frac{r^2 - 1}{2}}$$

$$\boxed{x = r}$$

$$x^2 = y - 1$$

$$\boxed{g^{-1}(1r) = r}$$

الف) $f = \{(0,0), (n,0), (n^2-n,0), (n,0), (-1,0)\}$ 14, 5 2

$m=2$ ✓ $n^2-n-2=0$ $\xrightarrow{-1 \times}$ $\downarrow$ 2 ✓

$n^2-n=2$ n=2 ✓

ب) $f = \{(-1,0), (1,0), (0,0), (n,0), (n^2-n,0), (0,0)\}$

m=2 ✓ a=-1 ✓ n=2 ✓

$f(x) = \begin{cases} 2x-1; & x \geq 1 \\ x+a; & x < 1 \end{cases}$

a < 1 $\leftarrow 1+a \leq 2$ ✓



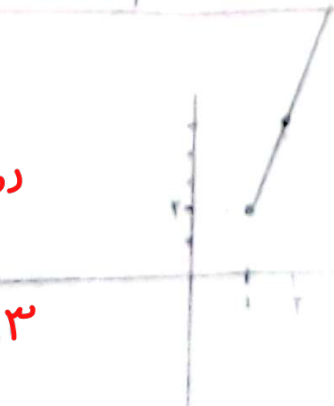
1, 7, 5 2

$f(x) = \begin{cases} 2x-1; & x \geq 1 \\ ax+a-1; & x \leq 0 \end{cases}$

دقت! a < 3 $\leftarrow 1 < 2+1$

I $a < -1$ $\rightarrow 2+1$ a < 3

II $a > 0$ I $\cap$ II $\rightarrow 0 < a < 3$



1, 7, 5

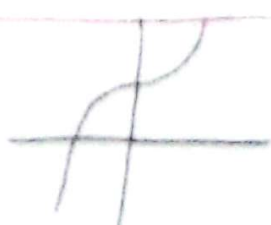
الف) $y = x^2 + 2$

$x_1^2 + 2 = x_2^2 + 2$

x₁ = x₂ ✓

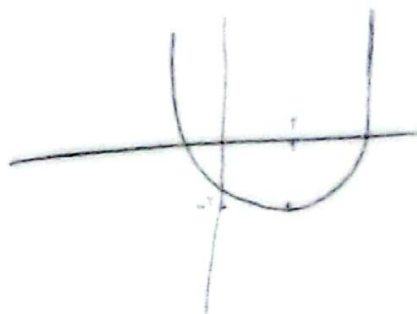
$x = y^2 + 1$

$y^2 = x - 2 \rightarrow y = \pm \sqrt{x-2}$ ✓



2

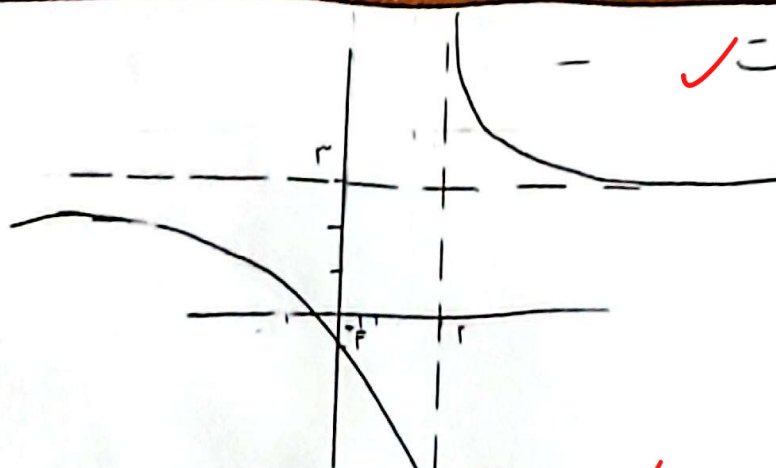
ب) $y = x^2 - 4x + 2$ ✓



$$y = \frac{r+1}{x-r}$$

یک به یک است ✓

(۲)



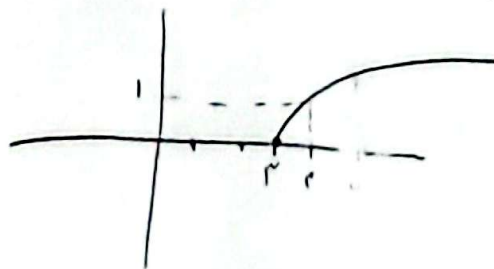
$$x = \frac{ry+1}{y-r} \rightarrow y = -\frac{-ry-1}{x-r} \rightarrow y = \frac{ry+1}{x-r} = f^{-1}$$

$$y = \frac{r^2+rx}{x+r} \rightarrow \frac{r(r+x)}{x+r} = r \rightarrow y = r$$

یک به یک نیست ✓

$$y = \sqrt{x-r} \rightarrow x = \sqrt{y-r} \rightarrow x^2 = y-r \rightarrow y = x^2 + r = f^{-1}$$

(۲)

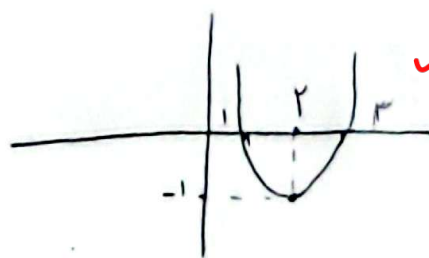


$$y = x^2 - rx + r \quad x \in [r, +\infty)$$

$$y = (x-r)^2 - 1 \rightarrow x = (y-r)^2 - 1$$

$$\overline{x-1} = y-r \leftarrow x+1 = (y-r)^2$$

$$\rightarrow y = r + \sqrt{x+1} \quad \checkmark$$



یک به یک است ✓

$$y = x + \sqrt{x} \quad y = k^2 + k \quad y(x+1) = k$$

$$\sqrt{x} = k \quad k = y^2 + y \quad y+1 = \frac{k}{y}$$

$$y = \frac{k}{y} + 1 \rightarrow y = \frac{k}{k^2+k} + 1 \rightarrow y = \frac{1}{k+1} + 1$$

$$\overline{x-1} \leftarrow \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}+1 \cdot \sqrt{x}-1} \leftarrow \frac{1+\sqrt{x}+1}{\sqrt{x}+1} \leftarrow y = \frac{1}{\sqrt{x}+1} + 1$$

$$f(x) = x + \sqrt{x} = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 - \frac{1}{x} \rightarrow f^{-1}(x) = \left(\sqrt{x + \frac{1}{x}} + \frac{1}{\sqrt{x}}\right)^2$$

$$f^{-1}(x) = x + \frac{1}{x} - \sqrt{x + \frac{1}{x}} \quad \begin{cases} a=1 \\ b=\frac{1}{x} \\ c=-\frac{1}{x} \end{cases} \rightarrow a+b+c=1$$