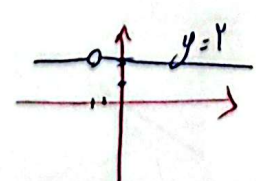
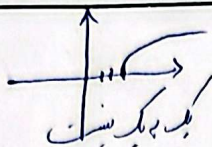


الف) $f = \{(-1, 4), (m, 2), (2, n^2 - n), (n, 5), (3, 2)\}$ $\Rightarrow m=3$ تابع یک به یک است $\Rightarrow n^2 - n = 2 \Rightarrow n^2 - n - 2 = 0 \Rightarrow n = -1$ یا $n = 2$ ب) $f = \{(-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 3)\}$ $\Rightarrow m=2$ تابع یک به یک $\Rightarrow f = \{(-1, 1), (1, 2), (2, 3), (a, 1), (a+2, n), (2, 3)\}$ $\Rightarrow a = -1$ $(1, n) \Rightarrow n=2$	۱
$f(x) = \begin{cases} 3x-1 & x \geq 1 \\ ax+a & x < 1 \end{cases}$ ۱) هر کدام از بازه ها باید یک به یک باشند ۲) اشتراک بردارها باید خالی باشند $y_1 = 3x-1 ; x \geq 1 \Rightarrow R_{y_1} = [2, +\infty)$ $y_2 = x+a ; x < 1 \Rightarrow R_{y_2} = (-\infty, a+1)$ $\Rightarrow [2, +\infty) \cap (-\infty, a+1) = \emptyset$ $\Rightarrow 2 \geq a+1 \Rightarrow a \leq 1$	۲
$f(x) = \begin{cases} 3x-1 & x \geq 1 \\ ax+a-1 & x < 0 \end{cases}$ شرط ۱) یک به یک هستند پس صیغ نباید همپوشانی شود. $a \neq 0$ $y_1 = 3x-1 ; x \geq 1 \Rightarrow R_{y_1} = [2, +\infty)$ $y_2 = ax+a-1 ; x < 0 \Rightarrow R_{y_2} = (-\infty, a-1]$ $\Rightarrow (-\infty, a-1] \cap [2, +\infty) = \emptyset$ $a-1 < 2 \Rightarrow a < 3$ صیغ y_2 باید نامنفی باشد $a > 0$ $0 < a < 3$	۳
الف) $y = x^3 + 2$ $y_1 = y_2 \Rightarrow x_1^3 + 2 = x_2^3 + 2 \Rightarrow x_1 = x_2$ یک به یک است $x^3 = y-2 \Rightarrow x = \sqrt[3]{y-2} \Rightarrow y = \sqrt[3]{x-2} \Rightarrow f^{-1}(x) = \sqrt[3]{x-2}$	۴
ب) $y = x^2 - 4x + 2$ صیغ است $\Rightarrow$ یک به یک نیست 	
الف) $y = \frac{3x+1}{x-2}$ یک به یک است $x = \frac{y+1}{y-3} \Rightarrow y = \frac{3x+1}{x-3} \Rightarrow f^{-1}(x) = \frac{3x+1}{x-3}$ ب) $y = \frac{4+2x}{x+2} = 2$  یک به یک نیست	۵

الف) $y = \sqrt{x-3}$



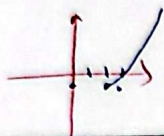
$y = \sqrt{x-3} \Rightarrow x-3 = y^2$

$\Rightarrow x = y^2 + 3 \Rightarrow y = \sqrt{x-3}$

$f^{-1}(x) = \sqrt{x-3} \leftarrow x \geq 3$

6

ب) $y = x^2 - 4x + 3, x \in [2, +\infty)$



یک به یک است

$y = x^2 - 4x + 3 \Rightarrow y = x^2 - 4x + 4 - 1 \Rightarrow (x-2)^2 - 1$

$\Rightarrow (x-2)^2 = y+1 \Rightarrow x = \pm \sqrt{y+1} + 2 \Rightarrow y = 2 \pm \sqrt{x+1}$

$\Rightarrow y = 2 + \sqrt{x+1} \quad f^{-1}(x) = \sqrt{x+1} + 2$

7

$y = x + \sqrt{x} \quad y_1 = y_2 \Rightarrow x_1 + \sqrt{x_1} = x_2 + \sqrt{x_2} \Rightarrow (x_2 - x_1) + (\sqrt{x_2} - \sqrt{x_1}) = 0$

$\Rightarrow (\sqrt{x_2} - \sqrt{x_1})(\sqrt{x_2} + \sqrt{x_1} + 1) = 0$

$\Rightarrow \sqrt{x_2} - \sqrt{x_1} = 0 \Rightarrow \sqrt{x_2} = \sqrt{x_1}$

$\sqrt{x_2} + \sqrt{x_1} + 1 \geq 1$

$\Rightarrow |x_2| = |x_1| \Rightarrow x_1 = x_2$

$y = x + \sqrt{x} \Rightarrow y = x + \sqrt{x} + \frac{1}{4} - \frac{1}{4} = (\sqrt{x} + \frac{1}{2})^2 - \frac{1}{4} \Rightarrow y + \frac{1}{4} = (\sqrt{x} + \frac{1}{2})^2 \Rightarrow \sqrt{x} + \frac{1}{2} = \pm \sqrt{y + \frac{1}{4}}$

$\Rightarrow \sqrt{x} = \pm \sqrt{y + \frac{1}{4}} - \frac{1}{2} \Rightarrow \sqrt{x} = \sqrt{y + \frac{1}{4}} - \frac{1}{2} \Rightarrow x = y + \frac{1}{4} - \sqrt{y + \frac{1}{4}} + \frac{1}{4} \Rightarrow y = x + \frac{1}{4} - \sqrt{y + \frac{1}{4}}$

$\Rightarrow f^{-1}(x) = x + \frac{1}{4} - \sqrt{x + \frac{1}{4}} \quad \left\{ \begin{array}{l} a=1 \\ b=\frac{1}{4} \\ c=-\frac{1}{4} \end{array} \right. \Rightarrow a+b+c = 1 + \frac{1}{4} - \frac{1}{4} = 1$

(V)

$y \neq \frac{x}{\sqrt{x^2-2}} \quad y_1 = y_2 \Rightarrow \frac{x_1}{\sqrt{x_1^2-2}} = \frac{x_2}{\sqrt{x_2^2-2}} \Rightarrow \frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2} \Rightarrow \frac{x_1^2}{-2} = \frac{x_2^2}{-2}$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

اما چون x و y هم علامت دارند و یک به یک است

$y = \frac{x}{\sqrt{x^2-2}} \Rightarrow y^2 = \frac{x^2}{x^2-2}$

$\Rightarrow x^2 = \frac{y^2}{y^2-1} \Rightarrow x = \pm \sqrt{\frac{y^2}{y^2-1}} \Rightarrow x = \frac{y\sqrt{y^2}}{\sqrt{y^2-1}} \Rightarrow y = \frac{x\sqrt{y^2}}{\sqrt{x^2-1}} \quad f^{-1}(x) = \frac{x\sqrt{y^2}}{\sqrt{x^2-1}}$

8

$g(x) = \sqrt{x-1} \Rightarrow g^{-1}(\sqrt{x-1}) = x \quad \sqrt{x-1} = -\frac{y}{\Delta} \Rightarrow \sqrt{x} = \frac{y}{\Delta} \Rightarrow x = \frac{y^2}{\Delta^2} \Rightarrow g^{-1}(\frac{y}{\Delta}) = \frac{y}{\Delta^2}$

$f^{-1}(x) = \frac{x}{1+|x|} \Rightarrow f(\frac{x}{1+|x|}) = x \quad \frac{x}{1+|x|} = -\frac{y}{\Delta} \Rightarrow \frac{x}{1-x} = -\frac{y}{\Delta} \Rightarrow \Delta x = 2x - y \Rightarrow 3x = -y \Rightarrow x = -\frac{y}{3}$

9

$\Rightarrow f(-\frac{y}{3}) + g^{-1}(-\frac{y}{3}) = \frac{y}{\Delta} - \frac{y}{3} = \frac{3y - \Delta y}{3\Delta} = \frac{-\Delta y}{3\Delta} = -\frac{y}{3} \quad f(-\frac{y}{3}) = -\frac{y}{3}$

$g(x) = f(x) + \sqrt{f(x)} \Rightarrow g^{-1}(\sqrt{f(x)} + f(x)) = x$

$f^{-1}(x) = \sqrt{x-1} \Rightarrow f(\sqrt{x-1}) = x$

$\sqrt{x-1} = t \Rightarrow t^2 = x-1 \Rightarrow x = t^2 + 1 \quad \left\{ \begin{array}{l} f(x) = x^2 + 1 \end{array} \right.$

$\Rightarrow g^{-1}(\sqrt{x-1} + x^2 + 1) = g^{-1}(12) \Rightarrow \sqrt{x-1} + x^2 + 1 = 12$

$\Rightarrow t^2 + t + 1 = 12 \Rightarrow t^2 + t - 11 = 0 \Rightarrow (t+4)(t-3) = 0 \Rightarrow t = 3 \Rightarrow \sqrt{x-1} = 3 \Rightarrow x-1 = 9 \Rightarrow x = 10$

10

$\Rightarrow g^{-1}(12) = 10$