

$$A = \{1, 2, 3\}$$

$$B = \{2, 3\}$$

$$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$B^c = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$A \times B - B^c = \{(1, 2), (1, 3)\}$$

الف) $f(n) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$

$(3, m^2), (3, m+2) \in f(n) \xrightarrow{n=m^2} f(n_1), f(n_2) \rightarrow m^2, m+2$

$m^2 - m - 2 = (m-2)(m+1) = 0 \rightarrow m=2$ ✓

ب) $g(n) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$

$(3, m^2), (3, m+2) \in f(n) \xrightarrow{n=m^2} g_1, g_2 \rightarrow m^2, m+2$

$m^2 - m - 2 = 0 \rightarrow m=2$ ✓

بازای $m=2$ می توانیم $g(m)$ تابع g متناهی بود.

$f(1) = 1 = a+b$

$f(2) = 4 = 2a+b$

$\begin{cases} a = 1 \\ b = -1 \end{cases} \rightarrow a \times b = -1$

$$f(n) = \begin{cases} n^2 - 1 & ; n \leq 1 \\ an + b & ; 1 \leq n \leq 2 \\ n^3 & ; n \geq 2 \end{cases}$$

$$f(n) = \begin{cases} 2n - 3 & ; n \geq k \\ 4 - 3n & ; n \leq k \end{cases}$$

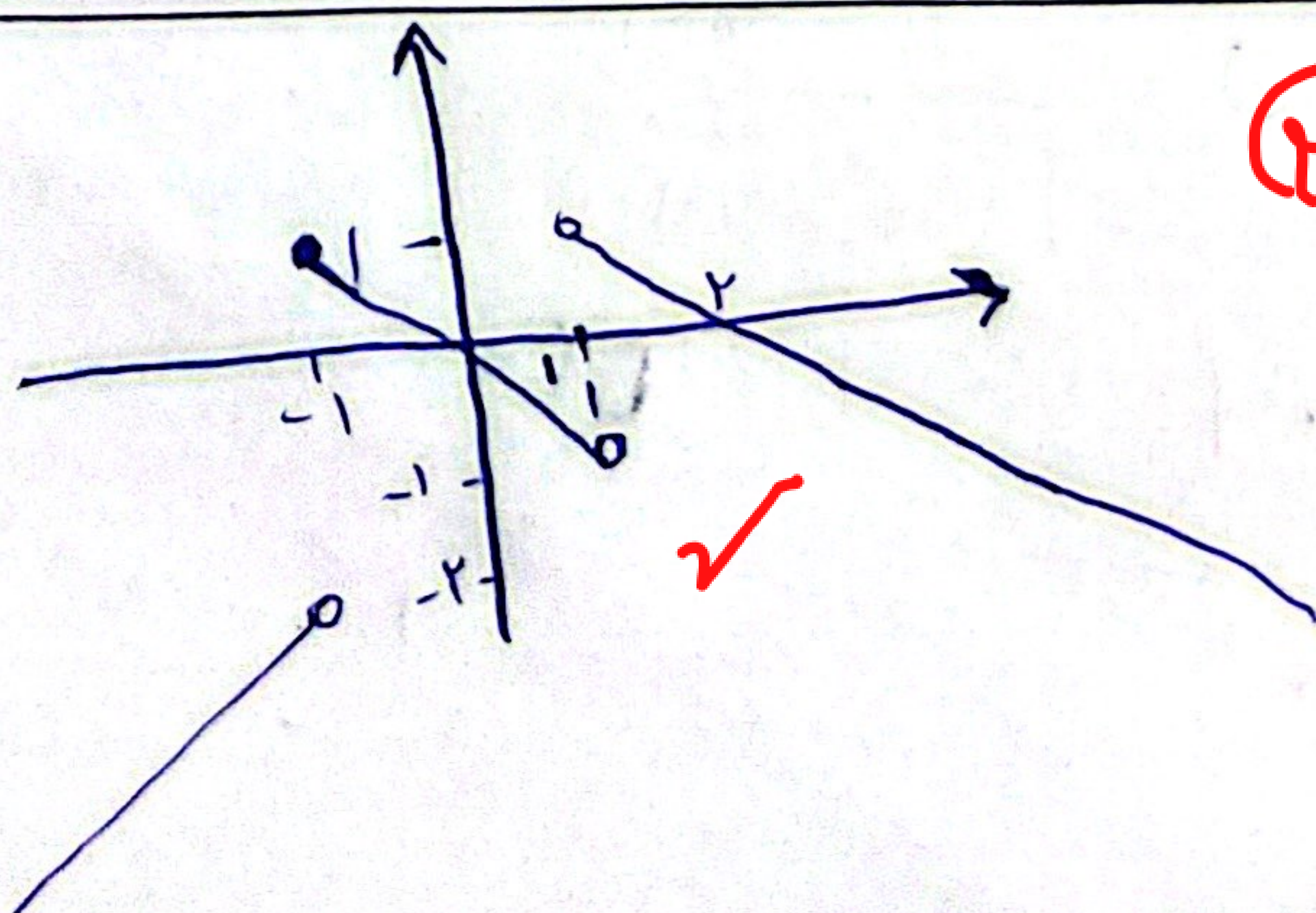
$f(k) = 2k - 3 = 4 - 3k$

$5k = 7 \rightarrow k = \frac{7}{5} = 1.4$

$$f(n) = \begin{cases} -n+2 & ; n > 1 \\ -n & ; -1 \leq n < 1 \\ n-1 & ; n < -1 \end{cases}$$

$$Df = \mathbb{R} - \{1\}$$

$$Rf = (-\infty, 1]$$



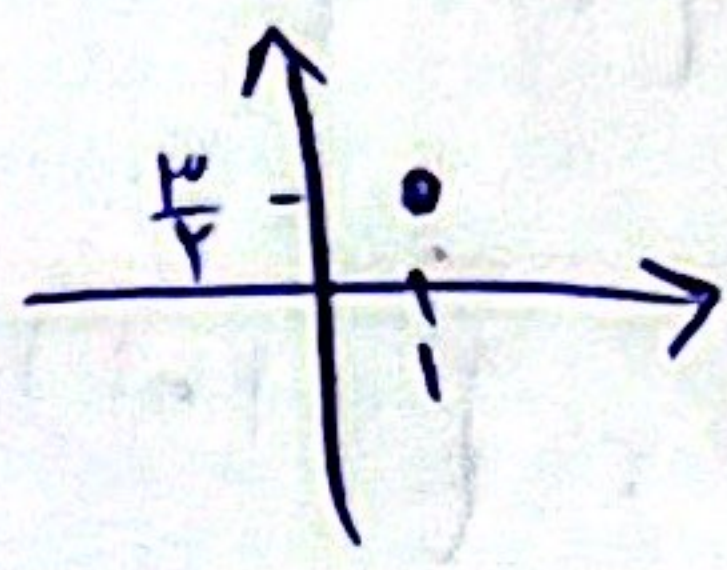
الف) $y^3 + x^3 + 1 = 0$

if $(x_1, y_1), (x_2, y_2) \in f \xrightarrow{x_1 = x_2} y_1 = y_2$

$y_1^3 + x_1^3 + 1 = 0 \quad y_2^3 + x_2^3 + 1 = 0 \xrightarrow{x_1 = x_2} y_1^3 = y_2^3$
 $y_1^3 = -x_1^3 - 1 \quad y_2^3 = -x_2^3 - 1 \xrightarrow{x_1 = x_2} y_1^3 = y_2^3$
 $y_1 = y_2$ تابع هسست ✓

ب) $x^3 + (y-1)^3 - 2x - 2y + 1 = 0$

$(x-1)^3 + (y-1)^3 = 0 \rightarrow$ تک نقطه $(1, 1) \rightarrow$ تابع هسست ✓



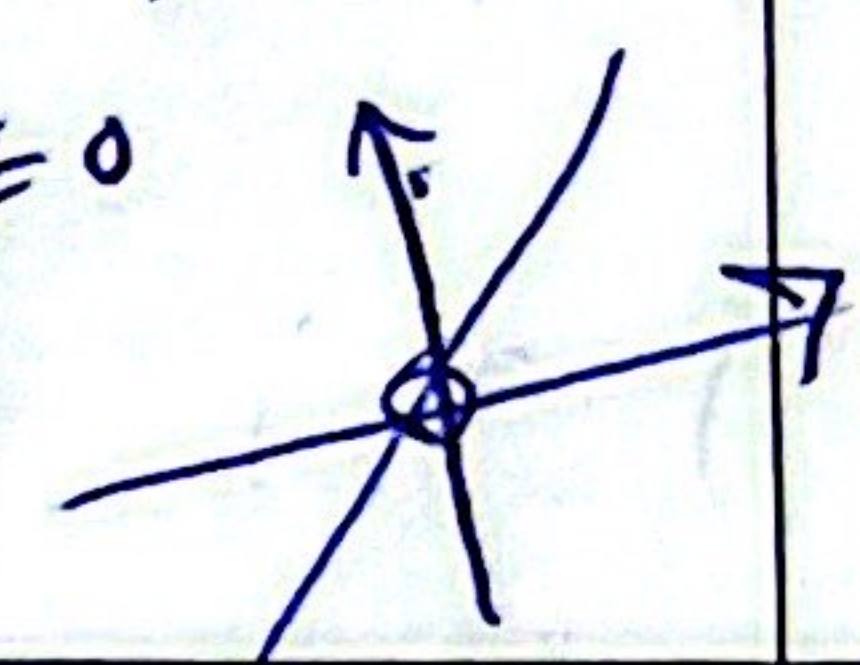
الف) $x \sin y = y \sin x \xrightarrow{x=0} 0 = y \sin 0$ مثال شش

$y \in \mathbb{R} \rightarrow$ به ازای $y \in \mathbb{R}$ مربعی می توان x را یافت ✓ تابع هسست ✓

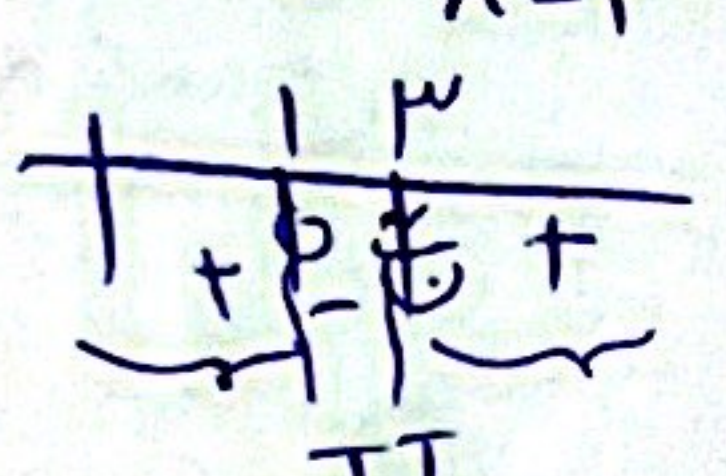
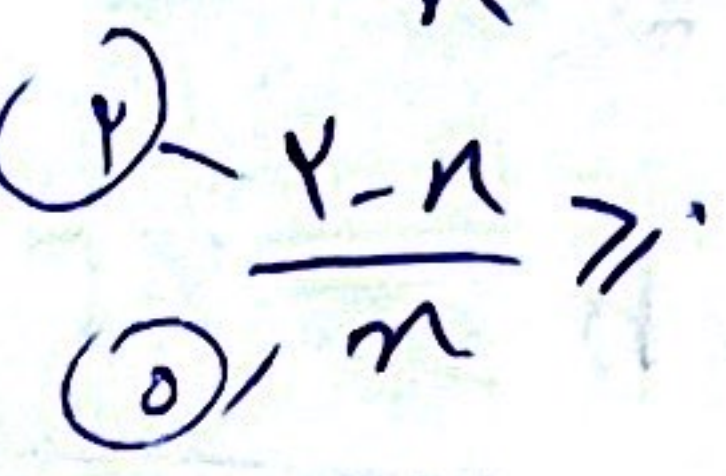
ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$

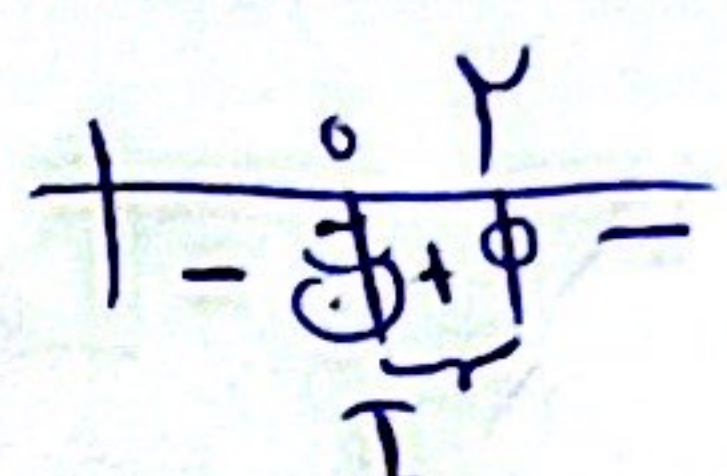
$\sqrt{\frac{x+y}{xy}} \cdot \sqrt{xy} = 2 \rightarrow \sqrt{x+y} = 2 \rightarrow x+y=4$

$\sqrt{\frac{x}{y}} = 2 - \sqrt{\frac{y}{x}} \rightarrow \frac{x}{y} = 4 - 4\sqrt{\frac{y}{x}} + \frac{y}{x} \rightarrow x = y$ ✓



الف) $y = \sqrt{\frac{n-1}{n-3}} + \sqrt{\frac{2-n}{n}}$

① $\frac{n-1}{n-3} \geq 0 \rightarrow$  $\frac{2-n}{n} \geq 0 \rightarrow$ 

② $\frac{2-n}{n} \geq 0 \rightarrow$  $D_f = (0, 1]$ ✓

ب) $\sqrt{x} + \sqrt{y-1} = \sqrt{3}$

$\sqrt{y-1} = \sqrt{3} - \sqrt{x} \rightarrow y-1 = 3 - 2\sqrt{x} + x \rightarrow y = x - 2\sqrt{x} + 4$

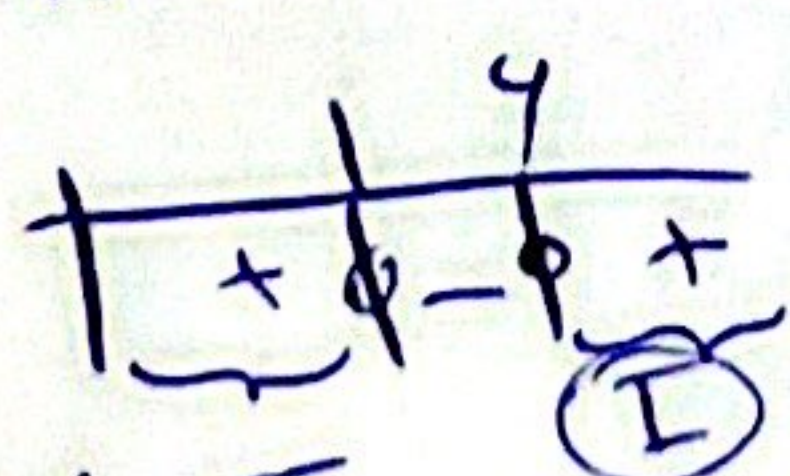
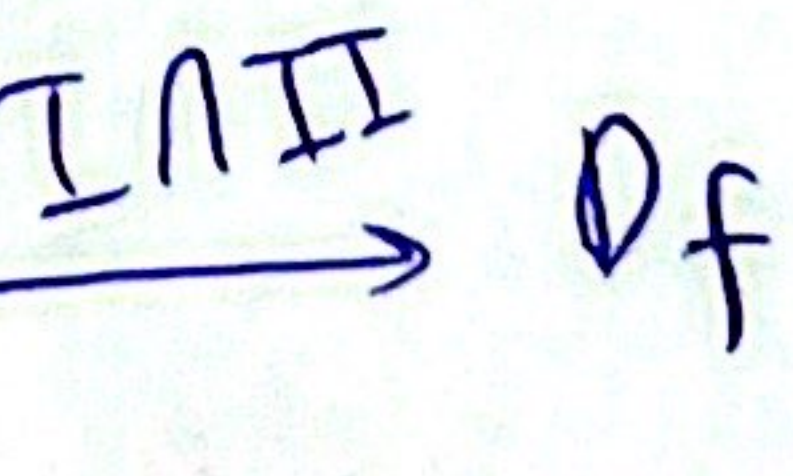
$\sqrt{y-1} \geq 0 \Rightarrow \sqrt{3} - \sqrt{x} \geq 0 \Rightarrow \sqrt{3} \geq \sqrt{x} \rightarrow 3 \geq x$

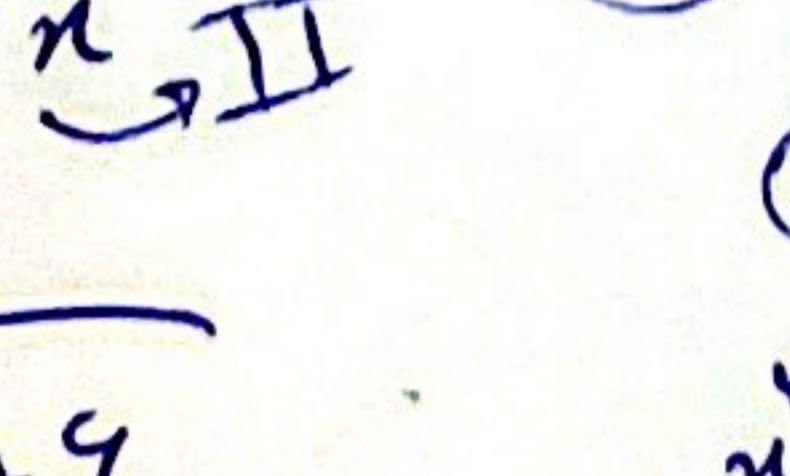
$\sqrt{y-1} \geq 0 \Rightarrow y-1 \geq 0 \Rightarrow y \geq 1$

$D_f = [0, 3]$ ✓

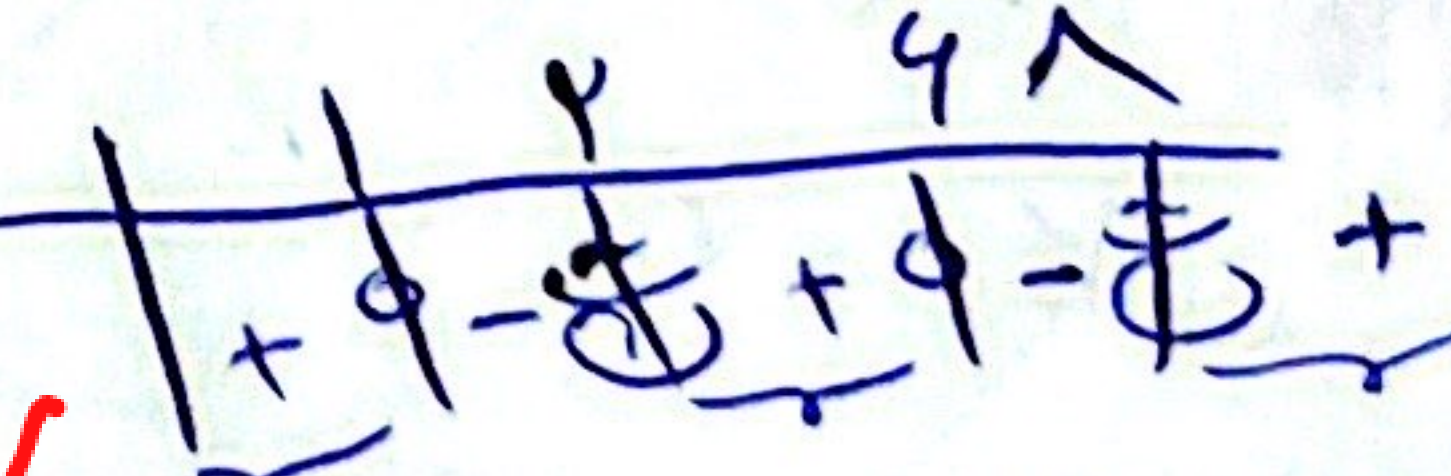
③ $D_f = \{x \geq 0, y \geq 1\}$

الف) $y = \frac{\sqrt{x^2 - \sqrt{x+9}}}{\sqrt{x^2 - 10x + 19}}$

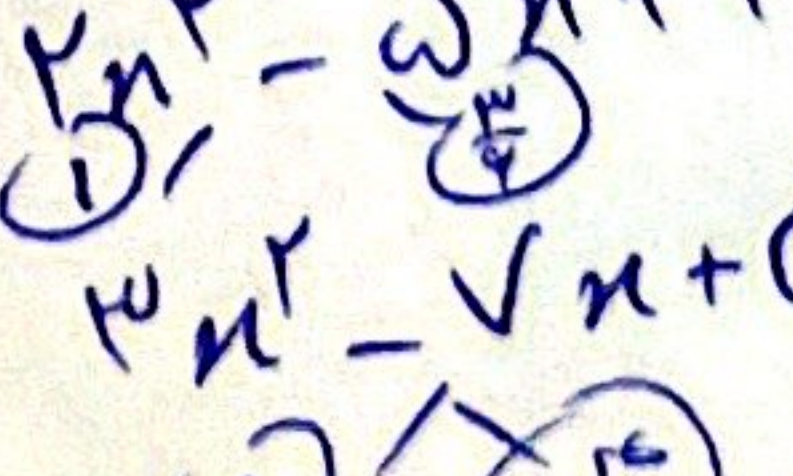
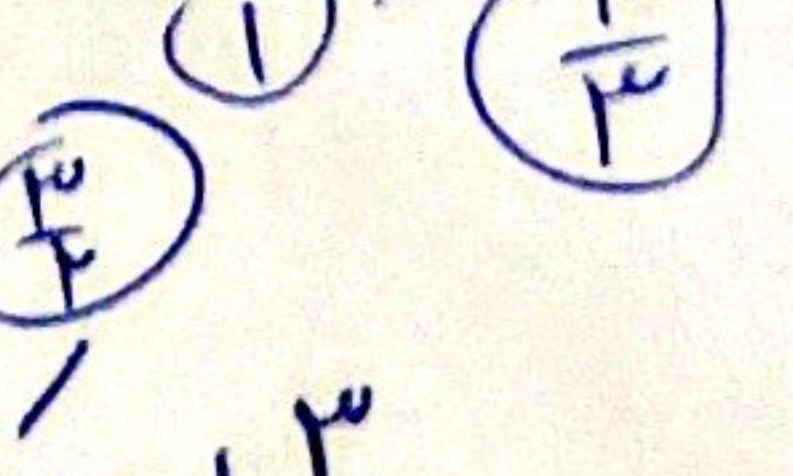
① $x^2 - \sqrt{x+9} \geq 0 \rightarrow$  $\frac{x^2 - \sqrt{x+9}}{x^2 - 10x + 19} \geq 0 \rightarrow$  $D_f = (-\infty, 1]$

② $-9x + 19 \geq 0 \rightarrow \frac{19}{9} \geq x \rightarrow$ 

ب) $y = \sqrt{\frac{x^2 - \sqrt{x+9}}{x^2 - 10x + 19}}$

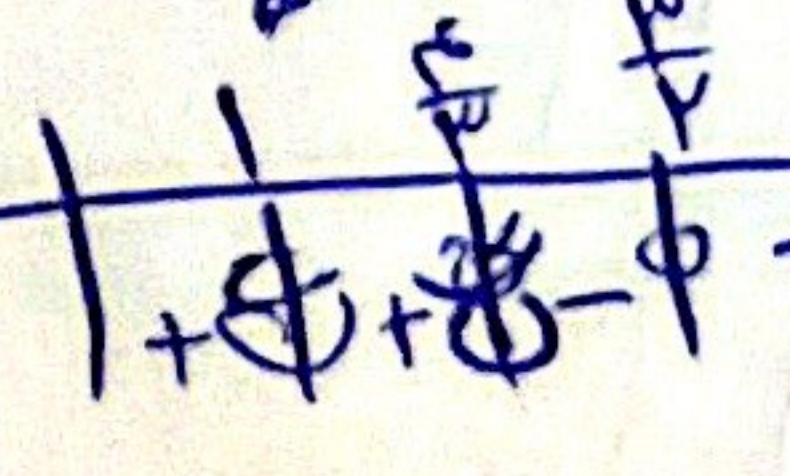
$\frac{x^2 - \sqrt{x+9}}{x^2 - 10x + 19} \geq 0 \rightarrow$  $D_f = (-\infty, 1] \cup (2, 2] \cup (10, +\infty)$

الف) $y = \frac{\sqrt{2x^2 - 5x + 3}}{\sqrt{3x^2 - \sqrt{x+9}}}$

$2x^2 - 5x + 3 \geq 0 \rightarrow$  $3x^2 - \sqrt{x+9} \geq 0 \rightarrow$ 

$I \cap II \rightarrow D_f = (-\infty, 1) \cup [\frac{3}{2}, +\infty)$ ✓

ب) $y = \sqrt{\frac{2x^2 - 5x + 3}{3x^2 - \sqrt{x+9}}}$

$\frac{2x^2 - 5x + 3}{3x^2 - \sqrt{x+9}} \geq 0 \rightarrow$  $D_f = (-\infty, 1) \cup (1, \frac{3}{2}) \cup [\frac{3}{2}, +\infty)$ ✓

$[\frac{3}{2}, +\infty)$