

① الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{x^2 - 2x + 1 - x^2}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}}$

Sign chart for $f(x)$:

Interval	Sign of $f(x)$
$(-\infty, 0)$	+
$(0, \frac{1}{2})$	-
$(\frac{1}{2}, 1)$	+
$(1, +\infty)$	-

$\therefore x = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}}$

Domain restrictions:

- $x \neq 1$
- $x \neq 0$
- $x \neq -2$
- $\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{3}{x-1} \neq \frac{-1}{x+2}$
- $3x+6 \neq 1-x$
- $4x \neq -5$
- $x \neq -\frac{5}{4}$

$D = \mathbb{R} - \{0, 1, -2, -\frac{5}{4}\}$

② ا) $\sqrt{\left(\left(\frac{1}{x}\right)^2 - 9\right)(3x-2)}$

Sign chart for $\sqrt{\left(\left(\frac{1}{x}\right)^2 - 9\right)(3x-2)}$:

Interval	Sign of $\sqrt{\left(\left(\frac{1}{x}\right)^2 - 9\right)(3x-2)}$
$(-\infty, -2)$	+
$(-2, \frac{2}{3})$	-
$(\frac{2}{3}, +\infty)$	+

$D = \mathbb{R} - (-2, \frac{2}{3})$

ب) $\sqrt[3]{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt[3]{x-1}$

آریتن سیردان/فر

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سرمه‌نده بابت بی نظمی در سراجا داده و در مسافر - نوشتم و نسخی که می‌خواهم نبود

$0 < \sqrt[3]{x-1} < 3$
 $1 < x-1 < 27$
 $2 < x < 28$

③ $f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x-1} + 1}$

Domain restrictions:

- $x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$
- $x-1 > 0 \rightarrow x > 1$

Sign chart for $f(x)$:

Interval	Sign of $f(x)$
$(-\infty, -1)$	+
$(-1, 2)$	-
$(2, +\infty)$	+

$D = (-\infty, -1) \cup (2, +\infty)$

③ $f(n) = \sqrt{n^2 + an - n^2} \xrightarrow[y=c]{n=-1} -1 - a + n^2 = 1$

$a + b = 1$

$a = -\frac{1}{2}$

$f(n) = \sqrt{-(n^2 + \frac{1}{2}n - n^2)}$

$p = -n \xrightarrow{\alpha = -1} \beta = b = \frac{n}{2}$

④ $g(n) = \sqrt{f(n) - n}$

$f(n) = \begin{cases} n^2 - 1 & n \geq 1 \\ n^2 + 1 & n < 1 \end{cases}$

$g(n) \begin{cases} n \geq 1 \rightarrow \sqrt{n^2 - 1 - n} = \sqrt{n^2 - 1} \rightarrow n \geq 1 \\ n < 1 \rightarrow \sqrt{n^2 + 1 - n} = \sqrt{n^2 + 1} \rightarrow n \geq -1 \end{cases}$

$-1 \leq n < 1$

$D = [-1, +\infty)$

⑤ $f(n) = n^2 - 5n = n(n - 5)$

$f(\sin \frac{\pi}{2} + \cos \frac{\pi}{2}) = f(\frac{\sqrt{2}}{2} + 1)$

$f(\frac{\sqrt{2}+1}{2}) = (\frac{\sqrt{2}+1}{2})(\frac{\sqrt{2}+1}{2} - 5) = \frac{1}{2}(\sqrt{2}+1)(\sqrt{2}-1) = -\frac{1}{2}$

$$\textcircled{v} \quad \left. \begin{aligned} f(n) &= \frac{n+1}{n-1} \\ f(f(a)) &= a \end{aligned} \right\} \rightarrow \frac{f(a)+1}{f(a)-1} = a$$

$$a f(a) - a = f(a)+1$$

$$f(a) = \frac{a+1}{a-1}$$

$$a \neq 1$$

$$D = \mathbb{R} - \{1\}$$

$$\textcircled{1} \quad \begin{aligned} f(n) &= n^r \cdot n \\ g(n) &= \sqrt{n-1} \\ f(g(a)) &= -r \end{aligned}$$

$$g'(n) - r g(a) = -r$$

$$g(a) = r$$

$$g(a) = r$$

$$\sqrt{a-1} = r \rightarrow a = \pm \sqrt{r^2 + 1}$$

$$\textcircled{9} \quad f(n+r) = n^r - 2n + r$$

$$f(r) \times f(-r) = r \times (-r) = -r^2$$

$$n=1 \downarrow \quad \downarrow n=-r$$

$$1-2r = \textcircled{r} \quad r^2 + r + r = \textcircled{\sqrt{r}}$$

$$\textcircled{10} \quad f(\sqrt{5}\cos^2 n + \Delta \cos n) = \tan^2 n$$

$$\hookrightarrow \sqrt{5}\cos^2 n + \Delta \cos n - \frac{\Delta}{\sqrt{5}} = 0$$

$$\rightarrow \cos n = \frac{-\Delta \pm \sqrt{\Delta^2 + 4\sqrt{5}}}{2\sqrt{5}}$$

$$\tan^2 n = \frac{1}{\cos^2 n} - 1$$

$$\rightarrow \cos n = \frac{-\Delta + 9}{7} = \frac{9}{7} = \frac{1}{1} \rightarrow \tan^2 n = 0$$

$$\rightarrow \frac{-\Delta - 9}{7} = \frac{-9}{7} \text{ etc.} \rightarrow -1, \cos 5, 1, -1$$