

$$\text{الف) } f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \Rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \quad (1)$$

$$\Rightarrow \frac{1-2x}{x(x-1)} \geq 0 \Rightarrow \frac{0}{-} \quad \frac{1}{+} \quad \frac{1}{-} \Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

$$\text{ب) } f(x) = \frac{1}{x+1} - \frac{x}{x} \Rightarrow x \neq -1, x \neq 0$$

$$\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow x \neq 1, x \neq -2$$

$$\frac{3x+9+x-1}{(x-1)(x+2)} \neq 0 \Rightarrow \frac{4x+8}{(x-1)(x+2)} \neq 0 \Rightarrow x \neq -\frac{2}{4} \Rightarrow D_f: \mathbb{R} - \{-2, -\frac{2}{4}, -1, 0, 1\}$$

$$\text{الف) } f(x) = \sqrt{\left[\left(\frac{1}{x}\right)^x - 9\right] (3x - x^x)} \Rightarrow \left(\left(\frac{1}{x}\right)^x - 9\right) (3x - x^x) \geq 0 \Rightarrow \begin{cases} 3x = x^x \\ = (x^x)^{\frac{1}{x}} \\ = x^{\frac{x}{x}} \end{cases} \quad (2)$$

$$\frac{-3}{+} \quad \frac{0}{-} \quad \frac{0}{+} \Rightarrow D_f = (-\infty, -3] \cup [\frac{0}{x}, +\infty)$$

$$\text{ب) } \sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{x-1} = 3 - \sqrt{y+1} \Rightarrow 3 - \sqrt{x-1} \geq 0$$

$$\Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$$

$$\Rightarrow D_f [1, 10]$$

$$f(x) = \log_{\frac{1}{2}} (x^2 - x - 2) \Rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow \frac{-1}{+} \quad \frac{2}{-} \quad \frac{2}{+} \quad \frac{-1}{-} \quad (3)$$

$$\frac{x^2-1}{\sqrt{x^2-1}+1} \Rightarrow x^2-1 \geq 0 \Rightarrow \frac{-1}{+} \quad \frac{1}{-} \quad \frac{1}{+} \Rightarrow D_f: (-\infty, -1) \cup (2, +\infty)$$

$$f(x) = \sqrt{3+ax - x^2} \quad D_f = [-2, b] \quad \text{برون است } x \text{ منفی است} \rightarrow \frac{-2}{-} \quad \frac{b}{+} \quad \frac{b}{-} \quad (4)$$

$$\Rightarrow \sqrt{3-2a-x} = 0 \Rightarrow \sqrt{-(2a+1)} = 0 \Rightarrow a = -\frac{1}{2} \Rightarrow f(x) = \sqrt{-x^2 - \frac{1}{2}x + 3}$$

$$\Rightarrow -x^2 - \frac{1}{2}x + 3 \geq 0 \Rightarrow -2x^2 - x + 6 \geq 0 \Rightarrow x^2 - x - 12 \geq 0 \Rightarrow (x-4)(x+3) \geq 0$$

$$\Rightarrow x \begin{cases} \frac{4}{-} \quad \frac{-3}{-} = -2 \\ \frac{-3}{+} \quad \frac{4}{+} = 7 \end{cases} = b \quad a+b = -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$$

$$f(u) = \begin{cases} ru - r & u \geq 1 \\ ru + r & u < 1 \end{cases}$$

$$g(u) = \sqrt{f(u) - u} \Rightarrow f(u) - u \geq 0$$

$$\begin{array}{lcl} & u \geq 1 & u < 1 \\ \swarrow & & \searrow \\ ru - r \geq 0 & & u + r \geq 0 \\ r(u - 1) \geq 0 & & u \geq -r \\ u \geq 1 & & \Downarrow \\ \Downarrow & & [-r, 1) \\ u \geq 1 & \cup & \\ [1, +\infty) & & \end{array}$$

$$\Rightarrow D_g = [-r, +\infty)$$

$$\begin{aligned} f(u) &= ru^r - \varepsilon u & f\left(\sin \frac{\pi}{r} + \cos r\pi\right) &= f\left(\frac{\sqrt{r}}{r} + 1\right) = f\left(\frac{r + \sqrt{r}}{r}\right) \\ \Rightarrow f(u) &= u(ru - \varepsilon) & &= \frac{r + \sqrt{r}}{r} (r + \sqrt{r} - \varepsilon) = \frac{r + \sqrt{r}}{r} \times \sqrt{r} - r = \frac{r - \varepsilon}{r} = \boxed{-\frac{1}{r}} \end{aligned}$$

$$f(u) = \frac{u+1}{u-1}$$

$$f(f(a)) = a \Rightarrow f\left(\frac{a+1}{a-1}\right) = a \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{\frac{ra}{a-1}}{\frac{-r}{a-1}} = a$$

$$\Rightarrow a = a$$

$\Rightarrow f(f(a)) = a$ به ازای هر مقدار معادله صحیح است.

$$f(u) = u^r - \varepsilon u$$

$$g(u) = \sqrt{u^r - 1}$$

$$f(g(a)) = -\varepsilon \Rightarrow (g(a))^r - \varepsilon g(a) = -\varepsilon$$

$$\Rightarrow [g(a)]^r - \varepsilon g(a) + \varepsilon = 0$$

$$\Rightarrow (g(a) - r)^r = 0 \Rightarrow g(a) = r \Rightarrow \sqrt{a^r - 1} = r$$

$$\Rightarrow a^r - 1 = r \Rightarrow \boxed{a = \pm \sqrt[r]{r+1}}$$

$$f(u+r) = u^r - \varepsilon u + r$$

$$f(u+r) = (u-r)(u-r)$$

$$f(\varepsilon) \times f(-r) = f(1+r) \times f(-r+r)$$

$$= -r \times (-1) \times (-r) \times (-1) = \boxed{1 \varepsilon \varepsilon}$$

$$f(\varepsilon \cos^r \eta + \delta \cos \eta) = \tan^r$$

$$\frac{1}{\cos^r \eta} - 1$$

$$f(r, \delta) = \frac{1}{\left(\frac{1}{r}\right)^r} - 1 = \frac{\frac{1}{r}}{\frac{1}{r}} - 1$$

$$\varepsilon - 1 = \boxed{r}$$

$$\varepsilon \cos^r \eta + \delta \cos \eta - r, \delta = 0$$

$$\varepsilon t^r + \delta t - r, \delta = 0 \Rightarrow t^r + \delta t - 1\varepsilon = 0 \Rightarrow (t + v)(t - r) = 0 \Rightarrow \cos \eta = t$$

$$\frac{r}{\varepsilon} = \frac{1}{r}$$

$$\frac{r}{\varepsilon} = \frac{1}{r}$$

$$-1 \leq \cos \eta \leq 1$$