نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۹ کلاس .....

۱

$f(f(\frac{\pi}{4})) : f(\cot\frac{\pi\times\pi}{4}) = f(\cot\frac{\pi}{4}) = f(\sqrt{2}) = \sqrt{(\sqrt{2})^2+1} = \sqrt{2+1} = \sqrt{3}$

$$f(n) = \begin{cases} \cot\frac{n\pi}{4} & ; n \le 1 \\ \sqrt{n^2+1} & ; n > 1 \end{cases}$$

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الف) $fog(\frac{\pi}{4}) = f(g(\frac{\pi}{4})) = 0$

$g(\frac{\pi}{4}) = 2\cos^2(\frac{\pi}{4}) = \frac{1}{2}$

$f(2) = \sqrt{8\times\frac{1}{4}-1} = 0$

ب) $fog(\sqrt{2}) = f(g(\sqrt{2}))$

$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}}$

$f(\frac{\sqrt{2}}{1-\sqrt{2}})$

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$gof(\frac{\pi}{4}) = g(f(\frac{\pi}{4})) \Rightarrow f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}} = \frac{1}{2}$

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الف) $fog(x) : \{(4,5)(2,5)(4,12)(1,12)\}$

$4 \to 4 \to 5$
$2 \to 4 \to 5$
$4 \to 1 \to 12$
$1 \to 6 \to 12$

ب) $gof(x) : \emptyset$

$4 \to 5$
$4 \to 5$
$1 \to 12$
$6 \to 12$

ج) $fof(x) : \emptyset$

$4 \to 5$
$4 \to 5$
$1 \to 12$
$6 \to 12$

د) $gog(x) : \{(4,1)(2,4)(4,6)\}$

$4 \to 4 \to 1$
$2 \to 4 \to 4$
$4 \to 1 \to 6$
$1 \to 6 \to \times$

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$fog : \{(1,1)(4,7)(a,2)(b,7)\}$

$a = 4$

$gof : \{(2,2)(4,1)\}$

$4 \to 5 \to 1$

$b = 5$

$(a,b) = (4,5)$

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$f(x) = ax + b \Rightarrow fof: a^2x + ab + b = 4x + 3 \rightarrow a = 2 \rightarrow b = 1$

$\qquad \rightarrow a = -2 \rightarrow b = -3$

$f(x) = \boxed{2x+1}$ یا $\boxed{-2x-3}$

$g(x) = cx + d \Rightarrow g(2x+3) = 2xc + 3c + d = 3x - 2 \rightarrow c = \frac{3}{2} \,/\, \frac{9}{2} + d = -2 \rightarrow d = -\frac{13}{2}$

$g(x) = \frac{3}{2}x - \frac{13}{2}$, $\quad f(-1) = -2 + 1 = -1$

$g(f(-1)) \Rightarrow g(-1) = -\frac{3}{2} - \frac{13}{2} = -\frac{16}{2} = -8$

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$$g(f(x)) = \frac{1}{x + |x| - 4\left(\sqrt{x + |x|}\right)}$$

$x + |x| - 4\left(\sqrt{x+|x|}\right) \neq 0$

$x + |x| \neq 4\sqrt{x+|x|}$

$(x + |x|)^2 \neq 16(x + |x|)$

$x + |x| \neq 16 \rightarrow D_f : \mathbb{R} - \{8\}$

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$(f+g)of \qquad D_{f+g} : f \cap g =$

$D_{(f+g)of} = \{x \in D_f : f(x) \in (D_g \cap D_f)\}$

$D_f : -1 \le x \le 1$

$D_g : x \ge 0$

$D_{(f+g)of} = \{-1 \le x \le 1 : 0 \le \sqrt{1-x^2} \le 1\}$ (لذا) $-1 \le x \le 1$

$\boxed{D_{(f+g)of} = -1 \le x \le 1}$

۹

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x + 5$

$t = \frac{3x+1}{x-2} \rightarrow \frac{-2t-1}{3-t} = x$

$f(t) = 4\left(\frac{-2t-1}{3-t}\right) + 5$

$f(t) = \frac{-8t-4}{3-t} + 5$

$f(x) = \frac{-8x-4}{3-x} + 5$

ب) $f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$

$f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 - 3x - \frac{3}{x}$

$f(x) = x^3 - 3\left(x + \frac{1}{x}\right)$

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تابع g به ازای ۱ و ۲√۲ صفر شود، پس برای آنکه $gof(x) = 0$ باید $f(x)$ برابر با ۱ و ۲√۲ باشد:

$f(x) = 1 \overset{x\sqrt{x}}{\Longrightarrow} x\sqrt{x} = 1 \Rightarrow x = 1$

$f(x) = 2\sqrt{2} \overset{x\sqrt{x}}{\Longrightarrow} x\sqrt{x} = 2\sqrt{2} \Rightarrow x = 2$

اختلاف نقاط: $\Delta x = 2 - 1 = 1$