

$$f\left(f\left(\frac{1}{\sqrt{2}}\right)\right): f\left(\cot \frac{\pi \times \frac{1}{\sqrt{2}}}{\sqrt{2}}\right) = f\left(\cot \frac{\pi}{2}\right) = f(0) = \sqrt{0^2 + 1} = \sqrt{1} = 1 \quad \checkmark$$

$$f(n) = \begin{cases} \cot \frac{\pi n}{\sqrt{2}} & ; n \leq 1 \\ \sqrt{n^2 + 1} & ; n > 1 \end{cases}$$

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$$f \circ g\left(\frac{1}{\sqrt{2}}\right) = f\left(g\left(\frac{1}{\sqrt{2}}\right)\right) = 0$$

$$g\left(\frac{1}{\sqrt{2}}\right) = 2 \cot\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} \quad \checkmark$$

$$f(1) = \sqrt{1^2 + 1} = \sqrt{2} \quad f\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$$

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$$f \circ g(\sqrt{2}) = f(g(\sqrt{2}))$$

$$g(\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} \quad g(\sqrt{2}) = -\sqrt{2} - 2$$

$$f\left(\frac{\sqrt{2}}{1 - \sqrt{2}}\right) = f(-\sqrt{2} - 2) = -4$$

$$g \circ f\left(\frac{\pi}{4}\right) = g\left(f\left(\frac{\pi}{4}\right)\right) \Rightarrow f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{2}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2} \quad \checkmark$$

$$f \circ g(n): \{(1, 0), (2, 0), (3, 1), (4, 1)\} \quad \checkmark$$

$1 \rightarrow 0$
 $2 \rightarrow 0$
 $3 \rightarrow 1$
 $4 \rightarrow 1$

$$g \circ f(n): \emptyset \quad \checkmark$$

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$1 \rightarrow 0$
 $2 \rightarrow 0$
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$$g \circ g(n): \{(1, 1), (2, 1), (3, 2)\} \quad \checkmark$$

$1 \rightarrow 1$
 $2 \rightarrow 1$
 $3 \rightarrow 2$
 $4 \rightarrow 2$

$$f \circ g: \{(1, 1), (2, 1), (3, 2), (4, 2)\} \quad g \circ f: \{(1, 1), (2, 1)\}$$

$$a = 1$$

$1 \rightarrow 0 \rightarrow 1$
 $2 \rightarrow 0 \rightarrow 1$

$$(a, b) = (1, 0) \quad \checkmark$$

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$$f(n) = an + b \Rightarrow f \circ f: a^n + ab + b = r^n + \dots \Rightarrow a = r \rightarrow b = 1$$

$$f(n) = \boxed{rn+1} \quad \boxed{-rn-r} \quad \checkmark$$

$$\rightarrow a = -r \rightarrow b = -r$$

$$g(n) = cn + d \Rightarrow g(rn+r) = rnc + r + d = rn - r \rightarrow c = \frac{r}{r} = 1, d = -r - r = -2r$$

$$g(n) = \frac{r}{r}n - \frac{1r}{r} \rightarrow f(-1) = -r + 1 = (-1)$$

$$g(f(-1)) \Rightarrow g(-1) = -\frac{r}{r} - \frac{1r}{r} = -\frac{1r}{r} = (-1) \quad \checkmark$$

$$g(f(n)) = \frac{1}{n+|n| - r(\sqrt{n+|n|})}$$

$$n+|n| - r(\sqrt{n+|n|}) \neq 0$$

$$n+|n| \neq r\sqrt{n+|n|}$$

$$(n+|n|)^r \neq 1r(n+|n|)$$

$$n+|n| \neq 1r$$

$$D_f: \mathbb{R} - \{1\} \quad D_g = \mathbb{R} - \{1, r\}$$

$$D_{g \circ f} = (0, +\infty) - \{1\}$$

$$(f+g) \circ f$$

$$D_{f+g}: f \cap g =$$

$$D_f: -1 \leq n \leq 1$$

$$D_{(f+g) \circ f} = \{n \in D_f: f(n) \in (D_g \cap D_f)\}$$

$$D_g: n > r$$

$$D_{(f+g) \circ f} = \{-1 \leq n \leq 1: 0 \leq \sqrt{1-n^2} \leq 1\}$$

$$D_{(f+g) \circ f} = -1 \leq n \leq 1 \quad \checkmark$$

$$\text{الف) } f\left(\frac{r+1}{n-r}\right) = r + \Delta \quad x = \frac{rt+1}{t-r}$$

$$t = \frac{r+1}{n-r} \rightarrow \frac{-rt-1}{r-t} = n$$

$$f(t) = r\left(\frac{-rt-1}{r-t}\right) + \Delta \quad f(n) = \frac{13n-11}{n-r}$$

$$f(t) = \frac{-1t-r}{r-t} + \Delta$$

$$f(n) = \frac{-1n-r}{r-n} + \Delta$$

$$\text{ب) } f\left(n+\frac{1}{n}\right) = n^r + \frac{1}{n^r}$$

$$f\left(n+\frac{1}{n}\right) = \left(n+\frac{1}{n}\right)^r - r^n - \frac{r}{n}$$

$$f(n) = n^r - r\left(n+\frac{1}{n}\right)$$

$$f(n) = n^r - rn$$

تابع g از روی $\sqrt{2}$ و $\sqrt{3}$ معرّفی شده است. برای $g \circ f(n) = 0$ باید $f(n)$ برابر با 1 و $\sqrt{2}$ باشد:

$$f(n) = 1 \Rightarrow n\sqrt{n} = 1 \Rightarrow n = 1 \quad \checkmark$$

$$f(n) = \sqrt{2} \Rightarrow n\sqrt{n} = \sqrt{2} \Rightarrow n = 2 \quad \checkmark$$

$$\Delta n = 2 - 1 = 1 \quad \checkmark$$

اختلاف نقاط: