

$$f(x) = \begin{cases} \cot \frac{\pi}{x} & : x \leq 1 \\ \sqrt{x^2 - 1} & : x > 1 \end{cases}$$

$$\frac{1}{x} < 1 \rightarrow f\left(\frac{1}{x}\right) = \cot \frac{\pi}{\frac{1}{x}} = \cot \frac{\pi}{x} = \sqrt{x^2 - 1}$$

$$\sqrt{x^2 - 1} > 1 \rightarrow f(\sqrt{x^2 - 1}) = \sqrt{(\sqrt{x^2 - 1})^2 - 1} = \sqrt{x^2 - 1} = x$$

$$f \circ f\left(\frac{1}{x}\right) = ? = f(\sqrt{x^2 - 1}) = x$$

$$f\left(\frac{1}{x}\right) = \sqrt{\frac{x^2 - 1}{x^2}} \quad f\left(\frac{1}{x}\right) = \sqrt{\frac{x^2 - 1}{x^2}} = \frac{\sqrt{x^2 - 1}}{x}$$

$$g(x) = x \cos \frac{\pi}{x} \rightarrow g\left(\frac{\pi}{x}\right) = x \cos \frac{\pi}{\frac{\pi}{x}} = \frac{1}{x}$$

$$g \circ g\left(\frac{\pi}{x}\right) = f\left(\frac{1}{x}\right) = \frac{\sqrt{x^2 - 1}}{x}$$

$$f(x) = [x] \quad f(-x - \sqrt{x^2 - 1}) = [-x - \sqrt{x^2 - 1}] = -x$$

$$g(x) = \frac{x^2}{1 - x^2} \quad g(\sqrt{x^2 - 1}) = \frac{(\sqrt{x^2 - 1})^2}{1 - (\sqrt{x^2 - 1})^2} = \frac{x^2 - 1}{1 - x^2 + 1} = -x - \sqrt{x^2 - 1}$$

$$f \circ g(\sqrt{x^2 - 1}) = f(-x - \sqrt{x^2 - 1}) = -x$$

$$f(x) = \sin x \rightarrow f\left(\frac{\pi}{x}\right) = \sin \frac{\pi}{x} = \frac{\sqrt{x^2 - 1}}{x}$$

$$g(x) = x \sqrt{1 - x^2} \quad g\left(\frac{\sqrt{x^2 - 1}}{x}\right) = \frac{\sqrt{x^2 - 1}}{x} \sqrt{1 - \left(\frac{\sqrt{x^2 - 1}}{x}\right)^2} = \frac{\sqrt{x^2 - 1}}{x} \sqrt{\frac{1}{x^2}} = \frac{1}{x}$$

$$g \circ f\left(\frac{\pi}{x}\right) = \frac{1}{x} = g\left(\frac{\sqrt{x^2 - 1}}{x}\right) = \frac{1}{x}$$

$$f(x) = \{(e, \omega), (y, \omega), (n, \omega), (1, \omega)\} \quad g(x) = \{(e, y), (e, \omega), (y, n), (n, 1)\}$$

$$f \circ g = \{(e, \omega), (y, \omega), (y, \omega), (n, \omega)\}$$

$$g \circ f = \emptyset$$

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$$g \circ g = \{(y, y), (e, n), (y, 1)\}$$

$$f = \{(y, 1), (e, \omega), (e, \omega), (1, y)\} \quad g = \{(1, y), (e, 1), (a, e), (b, 1)\}$$

$$(e, y) \in f \circ g \\ (e, 1) \in g \circ f$$

$$\rightarrow f \circ g(e) = y \rightarrow f(g(e)) = y \rightarrow g(e) = y \rightarrow a = y$$

$$(y, \omega)$$

$$g \circ f(e) = 1 \rightarrow g(f(e)) = 1 \rightarrow \begin{cases} f(e) = b \rightarrow b = \omega \\ f(e) = y \quad X \end{cases}$$

$$f(f(n)) = \begin{cases} \varepsilon n + r \\ a(n+b) + b = \varepsilon n + r \\ a^2 n + ab + b = \varepsilon n + r \end{cases} \quad \begin{cases} a^r = r \rightarrow a = \pm r \\ b(a+1) = r \end{cases}$$

$$\Rightarrow \begin{cases} a = r & b = 1 \rightarrow f(n) = rn + 1 \\ a = -r & b = -r \rightarrow f(n) = -rn - r \end{cases}$$

$$g(-1) = r(-1) - r = -1$$

$$\hookrightarrow f(-1) = \begin{cases} -1 \\ -1 \end{cases} \rightarrow f(-1) = -1$$

$$g(f(-1)) = ? = g(-1) = -1$$

$$f(n) = \sqrt{n+|n|} \quad \begin{cases} n+|n| \geq 0 \rightarrow D: \mathbb{R} \\ \sqrt{n+|n|} \neq 0, \varepsilon \end{cases} \quad \begin{cases} n+|n| \neq 0 \rightarrow n \geq 0 \\ n+|n| \neq 14 \rightarrow n \neq 14 \end{cases}$$

$$g(n) = \frac{1}{n^r - \varepsilon n} \rightarrow Dg = \mathbb{R} - \{0, \varepsilon\}$$

$$D_{g \circ f} = ? \xrightarrow{\text{چون } f \text{ و } g \text{ در } n=0 \text{ و } n=\varepsilon \text{ تعریف نشده اند}} = (0, +\infty) - \{1\}$$

$$f(n) = \sqrt{1-n^2} \quad \begin{cases} 1-n^2 \geq 0 \rightarrow -1 \leq n \leq 1 \\ \text{داشته} \end{cases}$$

$$g(n) = \sqrt{n} \rightarrow n \geq 0$$

$$\left. \begin{matrix} \text{اشتراک} \\ 0 \leq n \leq 1 \end{matrix} \right\} \rightarrow \begin{cases} 0 \leq \sqrt{1-n^2} \leq 1 \\ 1-n^2 \leq 1 \\ n^2 \neq 0 \rightarrow \mathbb{R} \end{cases}$$

$$D(f+g)(f(n)) = [0, 1]$$

$$f\left(\frac{r(n+1)}{n-r}\right) = \varepsilon n + \omega \rightarrow f(t) = \frac{\Lambda t + \varepsilon}{t-r} + \omega$$

$$t = \frac{r(n+1)}{n-r} \rightarrow f(n) = \frac{\Lambda n + \varepsilon}{n-r} + \omega$$

$$-n = + \frac{r\varepsilon + \omega}{t-r}$$

$$f(n) = \frac{\Lambda n - 11}{n-r}$$

$$f\left(n + \frac{1}{n}\right) = n^r + \frac{1}{n^r} + r\left(n + \frac{1}{n}\right) - r\left(n + \frac{1}{n}\right)$$

$$f\left(n + \frac{1}{n}\right) = \left(n + \frac{1}{n}\right)^r - r\left(n + \frac{1}{n}\right)$$

$$f(n) = n^r - rn$$

$$f(n) = n\sqrt{n} \quad \begin{matrix} |r| & |0| & |1| \\ \text{برای } g \text{ و } f \end{matrix}$$

$$g(f(n)) = 0 \rightarrow f(n) = \begin{cases} 1 \rightarrow n\sqrt{n} \geq 1 \rightarrow n = 1 \\ \sqrt{n} \rightarrow n\sqrt{n} \leq \sqrt{n} \rightarrow n \leq 1 \end{cases}$$

$$\{n \text{ برای } g \circ f \rightarrow |r|, |1| \rightarrow \text{فاصله وصل} = r-1 = 1$$