

$$f \circ f \left(\frac{r}{s} \right) = f \left(f \left(\frac{r}{s} \right) \right) = f \left(\sqrt{\frac{r}{s}} \right) = r \Rightarrow f \circ f \left(\frac{r}{s} \right) = r \quad (1)$$

$$\text{مثلاً } \frac{1}{2} : n \leq 1 \Rightarrow f \left(\frac{1}{2} \right) = \sqrt{\frac{1}{2}} = \sqrt{2}$$

$$\sqrt{n+1} : n > 1 \Rightarrow f \sqrt{2} = \sqrt{2} = r$$

$$\text{انی) } f(g(n)) \Rightarrow g \left(\frac{n}{f} \right) = \frac{1}{f} \quad f \left(\frac{1}{f} \right) = \sqrt{\frac{f^2-1}{f^2}} = \frac{\sqrt{f^2-1}}{f} \quad (2)$$

$$\Rightarrow f(g(n)) \quad \frac{\sqrt{f^2-1}}{1-\sqrt{f^2-1}} = -\sqrt{f^2-1} \quad [-\sqrt{f^2-1}]$$

$$1 \leq \sqrt{f^2-1} < f \Rightarrow f^2 < \sqrt{f^2-1} < f^2 \Rightarrow -f^2 < -\sqrt{f^2-1} < -f^2 \Rightarrow [-\sqrt{f^2-1}] = (-f) \quad (3)$$

$$g(f(\frac{n}{f}))$$

$$f \left(\frac{n}{f} \right) = \sin \frac{n}{f} = \sqrt{\frac{n}{f}} \quad g \left(\frac{\sqrt{n}}{f} \right) = \frac{\sqrt{n}}{f} \sqrt{1 - \frac{1}{f}} = \frac{\sqrt{n}}{f} \times \frac{1}{\sqrt{f}} = \frac{1}{\sqrt{f}} \quad (4)$$

$$\text{انی) } f \circ g(n) = \{ (t, d), (r, d), (y, l), (l, l) \}$$

$$\begin{array}{cc} t \rightarrow r \rightarrow d & r \rightarrow t \rightarrow d \\ y \rightarrow l \rightarrow l & l \rightarrow y \rightarrow l \end{array}$$

$$\Rightarrow g \circ f(n) = \emptyset \quad t \rightarrow d \rightarrow x \quad y \rightarrow l \rightarrow x \quad l \rightarrow y \rightarrow x \quad l \rightarrow l \rightarrow x$$

$$2-) f \circ f(n) = \emptyset \quad t \rightarrow d \rightarrow x \quad y \rightarrow d \rightarrow x \quad l \rightarrow l \rightarrow x, \quad l \rightarrow y \rightarrow x$$

$$3) g \circ g(n) = \{ (t, l), (r, r), (y, l) \} \quad \begin{array}{cc} t \rightarrow r \rightarrow l & r \rightarrow t \rightarrow y \\ y \rightarrow l \rightarrow l & l \rightarrow y \rightarrow x \end{array}$$

$$f \circ g(n) = \{ (t, l), (r, r), (y, l), (l, r) \}$$

$$f \circ g(t, r) \subseteq f \circ g \quad (5)$$

$$\Rightarrow a \in \{$$

$$g \circ f(n) = \{ (t, l), (r, r), (y, d), (l, r) \}$$

$$\Rightarrow b \in d$$

$$g \circ f(a, b) \in g \circ f \quad (a, b) \in (t, d)$$

$$f(an+bn) = a^n + ab + b \leq n+1 \Rightarrow \begin{cases} a^n \leq 1 \Rightarrow a \leq 1 \\ ab+bn \leq n \Rightarrow b \leq 1 \\ a \leq n \Rightarrow b \leq n \end{cases} \quad (4)$$

$$\begin{matrix} a \leq 1 \Rightarrow \\ a \leq n \Rightarrow \end{matrix} f(n) \leq n+1 \Rightarrow f(-1) \leq -1$$

$$\begin{matrix} a \leq n \Rightarrow \\ a \leq n \Rightarrow \end{matrix} f(n) \leq n+1 \xrightarrow{n \leq -1} f(-1) \leq -1 \Rightarrow f(-1) \leq -1$$

$$\Rightarrow g \circ f(-1) = -1$$

$$g(f(-1)) \Rightarrow g(-1) \Rightarrow g(-1) = -1$$

$$n+1 \leq -1 \Rightarrow n \leq -2$$

$$D_{g \circ f} = \{n \mid n \in D_f ; f(n) \in D_g\}$$

$$n \in \mathbb{N}, \quad \text{at } n=0$$

$$|n| \geq -n \Rightarrow \text{if } n \leq 0$$

$$g(n) = \frac{1}{n-n} \quad n \neq n \Rightarrow D_g = \mathbb{R} - \{0\}$$

$$n \in \mathbb{N} \neq 0 \Rightarrow |n| \neq -n \Rightarrow n > 0$$

$$\begin{aligned} \sqrt{n+|n|} &\neq 0 \quad n \neq -1 \\ n \in \mathbb{N} \neq 1 &\Rightarrow n \neq -1 \end{aligned}$$

$$\Rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$$

$$D_{(f \circ g) \circ f} = \{n \mid n \in D_g ; f(n) \in D_{f \circ g}\}$$

$$n \in [-1, 1]$$

$$D_{f \circ g} = D_f \cap D_g = [-1, 1]$$

$$g(n) = \sqrt{n} \Rightarrow D_g = [0, +\infty)$$

$$f(n) = \sqrt{n} \Rightarrow D_f = [-1, 1]$$

$$D_{f \circ g} \Rightarrow \begin{cases} -1 \leq \sqrt{n} \leq 1 \\ \sqrt{n} \leq 1 \\ n \leq 1 \end{cases} \Rightarrow n \in \mathbb{R}$$

$$\Rightarrow D_{(f \circ g) \circ f} = [-1, 1]$$

$$\begin{aligned} f\left(\frac{n+1}{n}\right) &\leq n+1 \\ \frac{n+1}{n-1} &\leq n \Rightarrow \frac{n+1}{n-1} \leq n \end{aligned} \Rightarrow f(n) = \left(\frac{n+1}{n-1}\right) - 1 = \frac{n+1-1}{n-1} = \frac{n}{n-1}$$

$$\therefore f\left(n + \frac{1}{n}\right) \leq n + \frac{1}{n} \Rightarrow f\left(n + \frac{1}{n}\right) = \left(n + \frac{1}{n}\right) - \frac{1}{n} = n$$

$$\Rightarrow f(n) = n - \frac{1}{n}$$

$$\begin{cases} g(f(n)) \leq 0 \\ g(1) = 0 \\ g(n) \leq 0 \end{cases}$$

$$\Rightarrow \begin{cases} f(n) \leq 1 \Rightarrow n \sqrt{n} \Rightarrow n \leq 1 \Rightarrow n \leq 1 \\ f(n) \leq \sqrt{n} \Rightarrow n \leq n \Rightarrow n \leq 1 \Rightarrow n \leq 1 \end{cases}$$

$$n \leq 1$$