

11, 10

$$f(x) = \begin{cases} \cot \frac{\pi x}{2} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$\rightarrow f \circ f \left(\frac{1}{2} \right) = ?$$

(1)

2

$$\textcircled{I} \rightarrow f\left(\frac{1}{2}\right) \quad \frac{1}{2} \leq 1 \rightarrow f\left(\frac{1}{2}\right) = \cot \frac{\pi}{4} = \cot \frac{\pi}{4} = \cot 45^\circ = \frac{\cos 45^\circ}{\sin 45^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{2}$$

(2)

$$\textcircled{II} \rightarrow f \circ f \left(\frac{1}{2} \right) = f\left(f\left(\frac{1}{2}\right)\right) = f(\sqrt{2}) \quad \sqrt{2} > 1 \rightarrow f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3} = f \circ f \left(\frac{1}{2} \right)$$

$$\text{الف) } f\left(\frac{1}{x}\right) = \sqrt{\frac{x^2 - 1}{x^2}} \quad , \quad g(x) = 2 \cos^2 x \rightarrow f \circ g \left(\frac{\pi}{4} \right) = ?$$

(3)

$$f \circ g \left(\frac{\pi}{4} \right) = f\left(g\left(\frac{\pi}{4}\right)\right) \rightarrow \begin{cases} \textcircled{I} g\left(\frac{\pi}{4}\right) = g(45^\circ) = 2(\cos 45^\circ)^2 = 2\left(\frac{1}{\sqrt{2}}\right)^2 = 1 \\ \textcircled{II} f\left(\frac{1}{1}\right) = \sqrt{\frac{1^2 - 1}{1^2}} = \sqrt{0} = 0 = f \circ g \left(\frac{\pi}{4} \right) \end{cases}$$

$$\text{ب) } f(x) = [x] \quad , \quad g(x) = \frac{x}{x-1} \rightarrow f \circ g(\sqrt{2}) = ?$$

15

$$f \circ g(\sqrt{2}) = f\left(g(\sqrt{2})\right) \rightarrow \begin{cases} \textcircled{I} g(\sqrt{2}) = \frac{\sqrt{2}}{\sqrt{2}-1} \xrightarrow{\text{مضروب کنیم}} \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2+\sqrt{2}}{2-1} = 2+\sqrt{2} \\ \textcircled{II} f(2+\sqrt{2}) = [2+\sqrt{2}] = 2 = f \circ g(\sqrt{2}) \end{cases}$$

(4)

$$f(x) = \sin x \quad , \quad g(x) = x\sqrt{1-x^2} \rightarrow g \circ f \left(\frac{\pi}{2} \right) = ?$$

$$g \circ f \left(\frac{\pi}{2} \right) = g\left(f\left(\frac{\pi}{2}\right)\right) = \begin{cases} \textcircled{I} f\left(\frac{\pi}{2}\right) = f(90^\circ) = \sin 90^\circ = 1 \\ \textcircled{II} g(1) = 1 \cdot \sqrt{1-1^2} = 0 = g \circ f \left(\frac{\pi}{2} \right) \end{cases}$$

2

$$g(x) = \{(x, y) | (x, y) \in (1, 2) \times (0, 1)\} \quad , \quad f(x) = \{(x, y) | (x, y) \in (1, 2) \times (0, 1)\}$$

(5)

$$\text{الف) } f \circ g(x) = \{(x, y) | (x, y) \in (1, 2) \times (0, 1)\} \checkmark$$

$$\text{ب) } g \circ f(x) = \{\cancel{(x, y)} | \cancel{(x, y)} \in (1, 2) \times (0, 1)\} = \emptyset \checkmark$$

$$\text{ج) } f \circ f(x) = \{\cancel{(x, y)} | \cancel{(x, y)} \in (1, 2) \times (0, 1)\} = \emptyset \checkmark$$

11, 10

$$\text{د) } g \circ g(x) = \{\cancel{(x, y)} | \cancel{(x, y)} \in (1, 2) \times (0, 1)\} = \{(x, y) | (x, y) \in (1, 2) \times (0, 1)\} \checkmark$$

$$f = \{(1,1), (2,2), (3,3), (4,4)\}, g = \{(1,2), (2,1), (3,3), (4,4)\}$$

if $(1,2) \in f \circ g, (1,1) \in g \circ f$

$$\text{نحوه } (1,2) \in f \circ g \rightarrow \text{if } m=1 \rightarrow g(1)=2 \rightarrow f(2)=2 \rightarrow \square = 2 \rightarrow m=1 \rightarrow \square = 2$$

$$\text{نحوه } (1,1) \in g \circ f \rightarrow \text{if } n=1 \rightarrow f(1)=1 \rightarrow g(1)=2 \rightarrow \square = 2 \rightarrow n=1 \rightarrow \square = 1$$

$$\text{نتیجه } (a,b) = (1,2) \checkmark$$

$$f(f(m)) = f(m+1), g(g(n)) = g(n-1) \rightarrow g \circ f(-1) = ?$$

$$\textcircled{1} f(m) = am+b \Rightarrow f(f(m)) = f(am+b) = a(am+b) + b = a^2m + ab + b$$

$$\text{نحوه } f(f(m)) = f(m+1) \rightarrow a^2m + ab + b = f(m+1) \rightarrow a^2 = 1, ab + b = b(a+1) = 1$$

$$\Rightarrow \text{if } \begin{cases} a=1 \rightarrow b(1+1)=1 \Rightarrow b=1/2 \rightarrow f(m) = m + 1/2 \\ a=-1 \rightarrow b(-1+1)=1 \Rightarrow b=1/0 \rightarrow \text{undefined} \end{cases}$$

$$\rightarrow f(-1) = \begin{cases} \text{معتبر} \rightarrow 1(-1) + 1/2 = -1/2 \\ \text{معتبر} \rightarrow 1(-1) - 1/2 = -3/2 \end{cases} \rightarrow f(-1) = -1/2 \checkmark$$

$$\rightarrow g(-1) = \begin{cases} \text{نحوه } 1m+1 = -1 \Rightarrow m = -2 \Rightarrow g(-1) = 1(-2) - 1 = -3 = g \circ f(-1) \end{cases}$$

$$f(m) = \sqrt{m+1}, g(m) = \frac{1}{m^2-1} \rightarrow g \circ f = ? \quad \textcircled{V}$$

$$f(m) = \sqrt{m+1} \Rightarrow m+1 \geq 0 \rightarrow \begin{cases} m \geq -1 \rightarrow m \in \mathbb{R} \\ m < -1 \rightarrow \text{undefined} \end{cases} \rightarrow D_{f(m)} = \mathbb{R}$$

$$g(m) = \frac{1}{m^2-1} \Rightarrow m^2-1 \neq 0 \Rightarrow m \neq \pm 1 \rightarrow D_{g(m)} = \mathbb{R} - \{1, -1\}$$

$$\text{log } \{m \in D_{f(m)} \mid f(m) \in D_{g(m)}\} \rightarrow \begin{cases} m < -1 \rightarrow -m = |m| \rightarrow f(m) = \sqrt{-m} = \sqrt{|m|} \\ m \geq -1 \rightarrow m = |m| \rightarrow f(m) = \sqrt{m} = \sqrt{|m|} \end{cases}$$

$$\text{نحوه } D_{g \circ f} \neq \emptyset \rightarrow \sqrt{m+1} \neq 1 \text{ and } \sqrt{m+1} \neq -1 \rightarrow m \neq 0 \text{ and } m \neq -2$$

$$[0, -2) \cup (-2, 0) \cup (0, \infty)$$

$$\checkmark D_{f \circ g} = (0, 1) \cup (1, \infty)$$

$$f(x) = \sqrt{1-x^2}$$

$$g(x) = \sqrt{x} \rightarrow D(f+g) = ?$$

(1)

$$\rightarrow f(x) = \sqrt{1-x^2} \rightarrow 1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \rightarrow D_f = [-1, 1]$$

$$\rightarrow g(x) = \sqrt{x} \rightarrow x \geq 0 \Rightarrow D_g = [0, +\infty) \Rightarrow D(f+g) = D_f \cap D_g = [0, 1]$$

$$\rightarrow (f+g)(x) \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \Rightarrow D(f+g) = [0, 1]$$

$$\text{الف) } f\left(\frac{x+1}{x-2}\right) = x-2 \rightarrow \frac{x+1}{x-2} = t \Rightarrow x = \frac{t^2+1}{t-2}$$

(2)

$$\rightarrow f(t) = \frac{-1t^2+11}{t-2} \rightarrow f(t) = \frac{t^2+11}{t-2} \rightarrow f(t) = \frac{x(t+1)}{t-2} \rightarrow 0$$

$$\Rightarrow \frac{1t+2}{t-2} + \frac{8(t-2)}{t-2} = \frac{t^2-11}{t-2} \rightarrow f(x) = \frac{t^2-11}{t-2}$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\rightarrow x + \frac{1}{x} = t \rightarrow f(t) = t^2 - 2 \rightarrow f(x) = x^2 - 2$$

$$g(x) = (x-1)(x-\sqrt{x}) = x^2 + (-1-\sqrt{x})x + \sqrt{x} \rightarrow f(x) = x\sqrt{x} = \sqrt{x^3}$$

(3)

$$\text{نوار} \rightarrow \log \rightarrow g(f(x)) = 0 \rightarrow f(x) = 1 \vee f(x) = \sqrt{x}$$

نقشه

$$\begin{cases} \text{I) } \sqrt{x^3} = 1 \Rightarrow x = 1 \\ \text{II) } \sqrt{x^3} = \sqrt{x} \Rightarrow x = 1 \end{cases} \rightarrow x_1 = x_2 = 1 = 1$$