ریاضی - تمرین تکلیف ۹ کلاس دوشنبه ها

سوال ۱)

$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x<1\\ \sqrt{x^2+1} & ; x>1\end{cases}\qquad fof\left(\frac{2}{3}\right)=?\quad f\left(f\left(\frac{2}{3}\right)\right)$$

$$\frac{2}{3}<1 \Rightarrow f\left(\frac{2}{3}\right)=\cot\frac{\pi}{4}\left(\frac{2}{3}\right)=\cot\frac{\pi}{6}=\sqrt{3}$$

$$\underset{\sqrt{3}>1}{\overset{\sqrt{3}\simeq 1/7}{\Longrightarrow}} f(\sqrt{3})=\sqrt{(\sqrt{3})^2+1}=\sqrt{4}=2 \qquad \Rightarrow f\left(f\left(\frac{2}{3}\right)\right)=2$$

سوال ۲)

الف) $fog\left(\frac{\pi}{3}\right)=?$ مقدار $g(x)=2\cos^2 x$ و $f\left(\frac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}}$

$$fog\left(\frac{\pi}{3}\right)=f\left(g\left(\frac{\pi}{3}\right)\right) \Rightarrow g\left(\frac{\pi}{3}\right)=2\cos^2\frac{\pi}{3}=2\left(\frac{1}{2}\right)^2=2\times\frac{1}{4}=\frac{1}{2}$$

$$f\left(\frac{1}{2}\right)\Rightarrow\sqrt{\frac{2(2)-1}{(\frac{1}{2})^2}}=\sqrt{\frac{3}{\frac{1}{4}}}=\sqrt{12}=2\sqrt{3} \Rightarrow fog\left(\frac{\pi}{3}\right)=2\sqrt{3}$$

ب) $fog(\sqrt{2})=?$ $\quad g(x)=\frac{x}{1-x}$ و $f(x)=[x]$

$$fog(\sqrt{2})=f(g(\sqrt{2}))\Rightarrow g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}(1+\sqrt{2})}{-1}=\frac{2+\sqrt{2}}{-1}=-\sqrt{2}-2$$

$$f(-\sqrt{2}-2)=[-\sqrt{2}-2]\Rightarrow \begin{matrix}\sqrt{2}\simeq 1/4\\ -\sqrt{2}-2=-3/4\end{matrix}\Rightarrow[-3/4]=-4\Rightarrow fog(\sqrt{2})=-4$$

سوال ۳) $gof\left(\frac{\pi}{4}\right)$ مقدار $g(x)=x\sqrt{1-x^2}$ و $f(x)=\sin x$

$$gof\left(\frac{\pi}{4}\right)=g\left(f\left(\frac{\pi}{4}\right)\right)\Rightarrow f\left(\frac{\pi}{4}\right)=\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$\Rightarrow g\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}=\frac{\sqrt{2}}{2}\sqrt{\underbrace{1-\frac{1}{2}}_{\frac{1}{2}}}=\frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}}=\frac{1}{2}\qquad gof\left(\frac{\pi}{4}\right)=\frac{1}{2}$$

سوال ۴)

$$f(x)=\{(2,5),(6,5),(8,12),(10,12)\}\qquad g(x)=\{(4,6),(2,4),(6,8),(8,10)\}$$

الف) $fog(x)=f(g(x))$

$$\begin{matrix}4\to 6\to 5\\ 2\to 4\to 5\\ 6\to 8\to 12\\ 8\to 10\to 12\end{matrix}\Rightarrow\{(4,5),(2,5),(6,12),(8,12)\}$$

ب) $gof(x)=g(f(x))$

$$\begin{matrix}4\to 5\to \times\\ 6\to 5\to \times\\ 8\to 12\to \times\\ 10\to 12\to \times\end{matrix}\Rightarrow\varnothing$$

ج) $f\circ f(x) = f(f(x))$

$$\begin{aligned} 4 &\to 5 \to \times \\ 6 &\to 5 \to \times \\ 8 &\to 12 \to \times \\ 10 &\to 12 \to \times \end{aligned} \Rightarrow \emptyset$$

د) $g\circ g(x) = g(g(x))$

$$\begin{aligned} 4 &\to 6 \to 8 \\ 2 &\to 4 \to 6 \\ 6 &\to 8 \to 10 \\ 8 &\to 10 \to \times \end{aligned} \Rightarrow \{(4,8),(2,6),(6,10)\}$$

سوال ۵)

$f = \{(2,1),(3,2),(4,5),(1,7)\}$

$g = \{(1,2),(3,1),(a,3),(b,1)\}$

$(4,2)\in f\circ g \Rightarrow (4,2)\in f(g(4))$ $\quad 2 \leftarrow 3 \leftarrow 4 \Rightarrow a = 4$ $\quad (a,b) = (4,5)$

$(4,1)\in g\circ f \Rightarrow (4,1)\in g(f(4))$ $\quad 4 \to 5 \to 1 \Rightarrow b = 5$

سوال ۶)

$f(f(x)) = 4x+3$

$g(2x+3) = 3x-2$

$g\circ f(-1) = ?$

$$\begin{cases} f(x) = ax+b \Rightarrow a(ax+b)+b = 4x+3 \Rightarrow a^2x + ab + b = 4x+3 \\ g(x) = cx+d \Rightarrow c(2x+3)+d = 3x-2 \Rightarrow 2cx + 3c + d = 3x-2 \end{cases}$$

$$\Rightarrow \begin{cases} a = \pm 2 \Rightarrow \begin{cases} a=2 \quad 3b=3 \to b=1 \Rightarrow f(x) = 2x+1 \\ a=-2 \quad -b=3 \Rightarrow b=-3 \Rightarrow f(x) = -x-3 \end{cases} \\ 2c = 3 \Rightarrow c = \frac{3}{2} \Rightarrow 3(\frac{3}{2}) + d = -2 \Rightarrow d = -\frac{4}{2} - \frac{9}{2} = \frac{-13}{2} \Rightarrow g(x) = \frac{3}{2}x - \frac{13}{2} \end{cases}$$

$$g\circ f(-1) = g(f(-1)) \Rightarrow \begin{cases} g(2(-1)+1) = g(-1) = \frac{3}{2}(-1) - \frac{13}{2} = \frac{-16}{2} = -8 \\ g(-(-1)-3) = g(-2) = \frac{3}{2}(-2) - \frac{13}{2} = \frac{-19}{2} \end{cases}$$

سوال ۷)

$f(x) = \sqrt{x+|x|}$

$g(x) = \frac{1}{x^2-4x}$ $\Rightarrow x^2 - 4x = 0 \quad x(x-4) = 0 \quad x=0,\ x=4$

$g\circ f \to D_{g\circ f} = ?$

$\sqrt{x+|x|} = 0 \quad x+|x| = 0 \Rightarrow \mathbb{R}^- \cup \{0\}$

$\sqrt{x+|x|} = 4 \Rightarrow x+|x| = 4 \Rightarrow 2x = 4 \Rightarrow x = 2$

به ازای صفر و منفی ها به $0=4$ میرسیم، غیر قابل قبول است.

$D_f = \mathbb{R} - \mathbb{R}^- \cup \{0\} \cup \{2\}$

سوال ۸)

$f(x)=\sqrt{1-x^2}$

$g(x)=\sqrt{x}$

$D_{(f+g)\circ f}=?$

$(f+g)\circ f = f\circ f + g\circ f = f(f(x)) + g(f(x))$

$= \sqrt{1-\sqrt{1-x^2}} + \sqrt{\sqrt{1-x^2}}$

$1-x^2 \geq 0 \quad x^2 \leq 1 \Rightarrow |x| \leq 1 \Rightarrow -1 \leq x \leq 1 \quad (I)$

$1-\sqrt{1-x^2} \geq 0 \quad \sqrt{1-x^2} \leq 1 \quad 1-x^2 \leq 1 \quad x^2 \geq 0$ همواره برقرار

$D_{(f+g)\circ f} = [-1,1]$

سوال ۹)

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \qquad \frac{3x+1}{x-2} = t \qquad x = -\frac{-2t-1}{t-3} = \frac{2t+1}{t-3}$

$f(t) = 4\left(\frac{2t+1}{t-3}\right)+5 = \frac{8t+4+5t-15}{t-3} = \frac{13t-11}{t-3} \Rightarrow f(x) = \frac{13x-11}{x-3}$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right) = \left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2-3\right)$

$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2 \qquad \Rightarrow f(x) = (x)\left((x)^2-3\right)$

سوال ۱۰)

$g(2\sqrt{2})=0$

$g(1)=0 \qquad f(x)=x\sqrt{x}$

$g\circ f=0 \qquad g(f(x))=0$

$\begin{cases} g(1)=0 \\ g(2\sqrt{2})=0 \\ g(f(x))=0 \end{cases} \Rightarrow \begin{array}{l} f(x)=1 \Rightarrow x\sqrt{x}=1 \Rightarrow x_1=1 \\ f(x)=2\sqrt{2} \Rightarrow x\sqrt{x}=2\sqrt{2} \Rightarrow x_2=2 \end{array}$

$|x_2-x_1| = |2-1| = 1$