نام و نام خانوادگی ......علی صفی...... ۲۰ پاسخنامه تشریحی تکلیف شماره ۹...... کلاس یازدهم سپهر A......

۱	$f_{(x)}=\begin{cases}\cot\frac{\pi x}{4} & ; x<1\\ \sqrt{x^2+1} & ; x\geq 1\end{cases} \rightarrow f\circ f_{(\frac{2}{3})}=f\Big(\underbrace{f_{(\frac{2}{3})}}_{\cot\frac{\pi}{6}}\Big)=f_{(\sqrt{3})}$ $\rightarrow f_{(\sqrt{3})}=\sqrt{3+1}=2$	۲
۲	الف) $f_{(\frac{1}{x})}=\sqrt{\frac{2x-1}{x^2}}\ ,\ g_{(x)}=2\cos^2 x \rightarrow f\circ g\,(\frac{\pi}{3})=f\,(\frac{1}{2})$ $\xrightarrow{x=2} f_{(\frac{1}{2})}=\sqrt{\frac{2\times 2-1}{4}}=\sqrt{\frac{3}{4}}$ ب) $g_{(\sqrt{2})}=\frac{\sqrt{2}}{1-\sqrt{2}} \rightarrow f\circ g_{(\sqrt{2})}=\left[\frac{\sqrt{2}}{1-\sqrt{2}}\right]=-4$	۲
۳	$f_{(x)}=\sin x\ ,\ g_{(x)}=x\sqrt{1-x^2} \rightarrow g\circ f\,(\frac{\pi}{4})=g\,(f_{(\frac{\pi}{4})})$ $=g\,(\sin\frac{\pi}{4})=g\,(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}}=\frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}}=\frac{1}{2}$	۲
۴	$f_{(x)}=\{(4,5),(6,5),(8,12),(10,12)\}\quad g_{(x)}=\{(4,6),(2,4),(6,8),(8,10)\}$ الف) $f\circ g\,(x) \rightarrow \{(4,5),(2,5),(6,12),(8,12)\}$ ب) $g\circ f_{(x)}=\varnothing$ د) $g\circ g_{(x)}=\{(4,8),(2,6),(6,10)\}$ ج) $f\circ f_{(x)}=\varnothing$	۲
۵	$f=\{(2,1),(3,2),(4,5),(1,7)\}\quad g=\{(1,2),(3,1),(\underset{4}{a},3),(\underset{5}{b},1)\}$ $(3,2)\in f\circ g \rightarrow a\rightarrow 3\rightarrow 2 \rightarrow a=4$ $(4,1)\in g\circ f \rightarrow 4\rightarrow 5=b\rightarrow 1$ $\rightarrow (a,b)=(4,5)$	۲

$f(f(x)) = 4x + 3 \rightarrow a^2x + ab + b = 4x + 3 \rightarrow \begin{cases} a = 2 \rightarrow b = 1 \rightarrow f(x) = 2x + 1 \\ a = -2 \rightarrow b = -3 \rightarrow f(x) = -2x - 3 \end{cases}$

$\rightarrow f(-1) = -1$

$g(2x+3) = 3x - 2 \rightarrow 2x + 3 = t \rightarrow x = \frac{t-3}{2} \rightarrow g(x) = \frac{3}{2}x - \frac{9}{2} - 2$ ۶

$\rightarrow x = -1 \rightarrow g(x) = -8 \qquad \rightarrow g \circ f(-1) = -8$ ✓ 2

$f(x) = \sqrt{x + |x|} \qquad g(x) = \frac{1}{x^2 - 4x} \qquad D_{g \circ f} = ?$

$D_{g \circ f} = \{x \in D_f \ , \ f(x) \in D_g\} = \{x \in \mathbb{R} \wedge f(x) \in \mathbb{R} - \{0, 4\}\}$ ۷

$= \{x \in \mathbb{R} \wedge (x > 0 \quad x \neq 8)\} \rightarrow D_{g \circ f} = (0, +\infty) - \{8\}$ ✓ 2

$f(x) = \sqrt{1 - x^2} \qquad g(x) = \sqrt{x} \longrightarrow D_{(f+g) \circ f} = ?$

$D_{(f+g) \circ f} = \{x \in D_f \ , \ f(x) \in D_{(f+g)}\} \rightarrow \{-1 \leq x \leq 1 \wedge 0 \leq \sqrt{1-x^2} \leq 1\}$ 2 ۸

$\{-1 \leq x \leq 1 \wedge \sqrt{1-x^2} \leq 1\} \Rightarrow \{-1 \leq x \leq 1 \wedge 0 \leq 1 - x^2 \leq 1\} = \{-1 \leq x \leq 1 \wedge 0 \leq x^2 \leq 1\}$

$= \{-1 \leq x \leq 1 \wedge -1 \leq x \leq 1\} \rightarrow D_{(f+g) \circ f} = [-1, 1]$ ✓

$f\left(\frac{3x+1}{x-2}\right) = 4x + 5 \longrightarrow \frac{3x+1}{x-2} = t \rightarrow x = \frac{2t+1}{t-3} \rightarrow f(t) = \frac{8t + 4 + 5t - 15}{t - 3}$

$f(x) = \frac{13x - 11}{x - 3}$ ✓ 2

$f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} \rightarrow x + \frac{1}{x} = t \rightarrow f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x \times \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$ ۹

$\rightarrow f(t) = t^3 - 3t \rightarrow f(x) = x^3 - 3x$ ✓

$f(x) = x\sqrt{x} \qquad \begin{cases} g(1) = 0 \\ g(2\sqrt{2}) = 0 \end{cases} \longrightarrow g \circ f(x) = 0$

2

$\left.\begin{array}{l} \rightarrow f(x) = 1 \longrightarrow x = 1 \ ✓ \\ \\ \longrightarrow f(x) = 2\sqrt{2} \longrightarrow x = 2 \end{array}\right\} \quad 2 \ ✓ \ 1 = 1$ ۱۰