

$$f(x) = \begin{cases} \cot \frac{\pi}{x}; & x < 1 \\ \sqrt{x+1}; & x > 1 \end{cases} \Rightarrow f \circ f\left(\frac{1}{2}\right) = f\left(f\left(\frac{1}{2}\right)\right)$$

$$\Rightarrow f\left(\frac{1}{2}\right) = \cot \frac{\pi}{2} = 0 < 1 \Rightarrow f(0) = \sqrt{0+1} = 1$$

$$f \circ g\left(\frac{\pi}{2}\right) = f\left(g\left(\frac{\pi}{2}\right)\right) \Rightarrow 2 \cos \frac{\pi}{2} = 2 \times \frac{1}{2} = 1$$

$$\Rightarrow f\left(\frac{1}{2}\right) \xrightarrow{x=2} \sqrt{\frac{2}{2}-1} = \sqrt{\frac{2}{2}} = \frac{\sqrt{2}}{2}$$

$$f \circ g(\sqrt{2}) = f(g(\sqrt{2})) \Rightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \Rightarrow f\left(\frac{\sqrt{2}}{1-\sqrt{2}}\right) = \left[\frac{\sqrt{2}}{1-\sqrt{2}}\right] \rightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{1-2} = \frac{\sqrt{2}+2}{-1} = -\sqrt{2}-2$$

$$g \circ f\left(\frac{\pi}{2}\right) = g\left(f\left(\frac{\pi}{2}\right)\right) \Rightarrow \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2} = \frac{1}{2}$$

$$g(g(m))$$

$$f \rightarrow g \rightarrow \alpha$$

$$2 \rightarrow f \rightarrow g$$

$$4 \rightarrow \alpha \rightarrow 1$$

$$\alpha \rightarrow 1 \rightarrow \alpha$$

$$= \{(f, g), (g, f), (f, \alpha), (g, \alpha)\}$$

$$f(f(m))$$

$$f \rightarrow \alpha \rightarrow \alpha$$

$$g \rightarrow g \rightarrow \alpha$$

$$\alpha \rightarrow 1 \rightarrow \alpha$$

$$1 \rightarrow 1 \rightarrow \alpha$$

$$= \emptyset$$

$$g(f(m))$$

$$f \rightarrow \alpha \rightarrow \alpha$$

$$g \rightarrow \alpha \rightarrow \alpha$$

$$\alpha \rightarrow 1 \rightarrow \alpha$$

$$1 \rightarrow 1 \rightarrow \alpha$$

$$= \emptyset$$

$$f(g(m))$$

$$f \xrightarrow{g(m)} g \xrightarrow{f(m)} \alpha$$

$$2 \rightarrow f \rightarrow \alpha$$

$$4 \rightarrow \alpha \rightarrow 1$$

$$\alpha \rightarrow 1 \rightarrow 1$$

$$= \{(f, \alpha), (g, \alpha), (f, 1), (g, 1)\}$$

$$(f, 2) \in f \circ g \rightarrow f(g(m)) = 2 \rightarrow f(g(f)) = 2 \Rightarrow g(f) = 2 \Rightarrow a = f$$

$$(f, 1) \in g \circ f \rightarrow g(f(m)) = 1 \rightarrow g(f(f)) = 1 \Rightarrow f(f) = 1 \Rightarrow b = \alpha$$

$$\Rightarrow (a, b) = (f, \alpha)$$

$$f(f(n)) = \varepsilon n + r \xrightarrow{f(n) = an+b} a(an+b) + b = \varepsilon n + r \Rightarrow a^2 n + ab + b = \varepsilon n + r$$

$$\rightarrow a^2 n + b(a+1) = \varepsilon n + r \Rightarrow a = \pm r, b = \begin{cases} -r \rightarrow f(n) = r(n+1), f(n) = -rn-r \\ r \rightarrow f(n) = r(n+1), f(n) = -rn-r \end{cases}$$

$$g(rn+r) = rn-r \xrightarrow{g(n) = cn+d} c(rn+r) + d = rn-r \Rightarrow rcn + r(c+d) = rn-r$$

$$\Rightarrow c = \frac{r}{r} = 1, \frac{r}{r} + d = -1 \Rightarrow d = -\frac{1r}{r} \Rightarrow g(n) = \frac{r}{r}n - \frac{1r}{r}$$

$$\Rightarrow g(f(-1)) = f(-1) = -1 \rightarrow g(-1) = -\frac{1r}{r} = -1$$

$$D_{g \circ f} = \{ n \in D_f : f(n) \in D_g \}$$

$$D_f = \mathbb{R} \quad D_g = \mathbb{R} - [0, r]$$

$$\sqrt{n+1} \in D_g \Rightarrow \sqrt{n+1} \notin [0, r] \rightarrow n \notin \{ \dots, r^2 \} \Rightarrow D_{g \circ f} = (0, +\infty) - \{ n \}$$

$$D_{(f+g) \circ f} = \{ n \in D_f : f(n) \in D_{(f+g)} \}$$

$$D_f = [-1, 1] \quad D_{(f+g)} = D_f \cap D_g = [-1, 1] \cap [0, +\infty) = [0, 1]$$

$$f(n) \in D_{f+g} \Rightarrow \sqrt{1-n^2} < 1 \Rightarrow n = [-1, 1]$$

$$f\left(\frac{r(n+1)}{n-r}\right) = \varepsilon n + r \Rightarrow \frac{r(n+1)}{n-r} = t \Rightarrow n = \frac{r(t+1)}{t-r}$$

$$\Rightarrow f(t) = \frac{1t+r}{t-r} + r, f(n) = \frac{1n+r}{n-r} + r = \frac{1n-1}{n-r}$$

$$+ \leftarrow f\left(n + \frac{1}{n}\right) = n^r + \frac{1}{n^r} \Rightarrow n + \frac{1}{n} = t \rightarrow \left(n + \frac{1}{n}\right)^r = n^r + \frac{1}{n^r} + r n^{r-1} + \frac{r}{n^{r+1}}$$

$$+ \leftarrow \left(n + \frac{1}{n}\right)^r - r \left(n + \frac{1}{n}\right) = n^r + \frac{1}{n^r} \Rightarrow f(n) = n^r - r n$$

$$f(ay) \xrightarrow{\text{عز}} 1, 2\sqrt{r}$$

$$f(n) = n\sqrt{n}$$

$$g(f(n)) = 0 \rightarrow f(n) \rightarrow n\sqrt{n} = 1, 2\sqrt{r} \begin{cases} n\sqrt{n} = 1 \rightarrow n = 1 \\ n\sqrt{n} = 2\sqrt{r} \rightarrow n = r \end{cases}$$

$$[r-1, 1] \leftarrow$$