نام و نام خانوادگی پارسا جهانگیری ...... پاسخنامه تشریحی تکلیف شماره ۹ ..... کلاس یازدهم ریاضی A ......

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$fof(\frac{2}{3}) = f(f(\frac{2}{3})) = f(\sqrt{3}) = 2 \Rightarrow \boxed{fof(\frac{2}{3}) = 2}$

$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \le 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases} \Rightarrow \begin{cases} f(\frac{2}{3}) = \cot \frac{\pi}{6} = \sqrt{3} \\ f(\sqrt{3}) = \sqrt{4} = 2 \end{cases}$

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الف) $fog(\frac{\pi}{3}) = f(g(\frac{\pi}{3})) = f(\frac{1}{2}) = \frac{\sqrt{3}}{2} \qquad \boxed{fog(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}}$

$g(x) = 2\cos^2 x \Rightarrow g(\frac{\pi}{3}) = 2\cos^2 \frac{\pi}{3} = 2(\frac{1}{2})^2 = \frac{1}{2}$

$f(\frac{1}{x}) = \sqrt{\frac{2x-1}{x^2}} \xrightarrow{x=2} f(\frac{1}{2}) = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

ب) $fog(\sqrt{2}) = f(g(\sqrt{2})) = f(-2-\sqrt{2}) = -4 \qquad \boxed{fog(\sqrt{2}) = -4}$

$g(x) = \frac{x}{1-x} \xrightarrow{x=\sqrt{2}} g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = -(2+\sqrt{2}) = -2-\sqrt{2}$

$f(x) = [x] \xrightarrow{x=-2-\sqrt{2}} f(-2-\sqrt{2}) = [-2-\sqrt{2}] \xrightarrow{\sqrt{2}=1.4} [-3.4] = -4$

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$gof(\frac{\pi}{4}) = g(f(\frac{\pi}{4})) = g(\frac{\sqrt{2}}{2}) = \frac{1}{2} \qquad \boxed{gof(\frac{\pi}{4}) = \frac{1}{2}}$

$f(x) = \sin x \xrightarrow{x=\frac{\pi}{4}} f(\frac{\pi}{4}) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$g(x) = x\sqrt{1-x^2} \xrightarrow{x=\frac{\sqrt{2}}{2}} g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

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الف) $fog(x) = \{(4,5),(2,5),(6,12),(8,12)\}$

$4 \to 6 \to 5$
$2 \to 4 \to 5$
$6 \to 8 \to 12$
$8 \to 10 \to 12$

ب) $gof(x) = \emptyset$

$4 \to 5 \to \times$
$6 \to 5 \to \times$
$8 \to 12 \to \times$
$10 \to 12 \to \times$

ج) $fof(x) = \emptyset$

$4 \to 5 \to \times$
$6 \to 5 \to \times$
$8 \to 12 \to \times$
$10 \to 12 \to \times$

د) $gog(x) = \{(4,8),(2,6),(6,10)\}$

$4 \to 6 \to 8$
$2 \to 4 \to 6$
$6 \to 8 \to 10$
$8 \to 10 \to \times$

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$\boxed{(4,2) \in fog}$ (I)

$1 \to 2 \to 1$
$3 \to 1 \to 7$
$a \to 3 \to 2$
$b \to 1 \to 7$

$fog(x) = \{(1,1),(3,7),\boxed{(a,2)},(b,7)\}$ (II)

$(4,1) \in gof$

$2 \to 1 \to 2$
$3 \to 2 \to 3$
$4 \to b \to 1$ (III)
$1 \to 7$

$\xrightarrow{(I),(II)} (4,2) = (a,2) \Rightarrow \boxed{a=4}$ (*)

$f = \{(2,1),(3,2),\boxed{(4,5)},(1,7)\}$ (IV)

$\xrightarrow{(III),(IV)} (4,b) = (4,5) \Rightarrow \boxed{b=5}$ (**)

$\xrightarrow{(*),(**)} (a,b) = (4,5)$

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f تابعی خطی است پس آن را به فرم $f(x)=ax+b$ می نویسیم.

$f(f(x))=f(ax+b)=a^2x+ab+b=4x+3$ $\xrightarrow{\text{متحد قرار دادن}}$ $\begin{cases} a^2=4 \Rightarrow a=\pm 2 \\ ab+b=3 \end{cases}$ $\xrightarrow{a=2}$ $3b=3 \Rightarrow b=1$; $\xrightarrow{a=-2}$ $-b=3 \Rightarrow b=-3$

$f(x)=ax+b \xrightarrow{a=2,\,b=1} f(x)=2x+1 \xrightarrow{x=-1} f(-1)=-1$

$\xrightarrow{a=-2,\,b=-3} f(x)=-2x-3 \xrightarrow{x=-1} f(-1)=-1$ $\Rightarrow f(-1)=-1$

$gof(-1)=g(f(-1))=g(-1)=-8$

$g(3x+2)=3x-5 \xrightarrow[x=-1]{3x+2=-1} g(-1)=-8$

$\boxed{gof(-1)=-8}$ ✓ ۲

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$D_{gof}=\{x \mid x\in D_f \;;\; f(x)\in D_g\}$

$f(x)=\sqrt{x+|x|} \Rightarrow x+|x|\geq 0 \Rightarrow |x|\geq -x$ همواره برقرار است $\Rightarrow D_f=\mathbb{R}$

$g(x)=\dfrac{1}{x^2-4x} \Rightarrow x^2-4x\neq 0 \Rightarrow x(x-4)\neq 0 \Rightarrow x\neq 0,4 \Rightarrow D_g=\mathbb{R}-\{0,4\}$

$x\in\mathbb{R}$ (I)

$\begin{cases} \sqrt{x+|x|}\neq 0 \Rightarrow x+|x|\neq 0 \Rightarrow |x|\neq -x \Rightarrow x>0 \quad (II) \\ \sqrt{x+|x|}\neq 4 \Rightarrow x+|x|\neq 16 \Rightarrow 2x\neq 16 \Rightarrow x\neq 8 \quad (III) \end{cases}$

$(I)\cap(II)\cap(III) \Rightarrow \boxed{D_{gof}=(0,+\infty)-\{8\}}$ ✓ ۲

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$D_{(f+g)of}=\{x \mid x\in D_f \;;\; f(x)\in D_{f+g}\}$

$x\in[-1,1]$

$D_{f+g}=D_f\cap D_g \xrightarrow{(I),(II)} D_{f+g}=[0,1]$

$g(x)=\sqrt{x} \Rightarrow x\geq 0 \Rightarrow D_g=[0,+\infty)$ (I)

$f(x)=\sqrt{1-x^2} \Rightarrow 1-x^2\geq 0 \Rightarrow (1-x)(1+x)\geq 0 \rightarrow D_f=[-1,1]$ (II)

(III) $\begin{cases} \sqrt{1-x^2}\in[0,1] \\ \Rightarrow \sqrt{1-x^2}\leq 1 \Rightarrow 1-x^2\leq 1 \\ \Rightarrow x^2\geq 0 \Rightarrow x\in\mathbb{R} \quad (IV) \end{cases}$ همواره برقرار است

$\sqrt{1-x^2}\geq 0$ بدیهی

$(III)\cap(IV) \Rightarrow D_{(f+g)of}=[0,1] \rightarrow [-1,1]$

$f(x)\in D_{g+f}$

$0\leq\sqrt{1-x^2}\leq 1$

$x^2\geq 0$ همواره ✓

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الف) $f\left(\dfrac{3x+1}{x-2}\right)=4x+5$; $\dfrac{3x+1}{x-2}=t \Rightarrow \dfrac{2t+1}{t-3}=x$

$\Rightarrow f(t)=4\left(\dfrac{2t+1}{t-3}\right)+5=\dfrac{8t+4}{t-3}+5=\dfrac{13t-11}{t-3} \Rightarrow \boxed{f(x)=\dfrac{13x-11}{x-3}}$ ✓

ب) $f\left(x+\dfrac{1}{x}\right)=x^3+\dfrac{1}{x^3} \xrightarrow{x+\frac{1}{x}=t} f(t)=t^3-3t \Rightarrow \boxed{f(x)=x^3-3x}$ ✓

$x^3+\dfrac{1}{x^3}=\left(x+\dfrac{1}{x}\right)^3-3(x)\left(\dfrac{1}{x}\right)\left(x+\dfrac{1}{x}\right) \Rightarrow x^3+\dfrac{1}{x^3}=\left(x+\dfrac{1}{x}\right)^3-3\left(x+\dfrac{1}{x}\right)$

$\xrightarrow{x+\frac{1}{x}=t} x^3+\dfrac{1}{x^3}=t^3-3t$

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$\begin{cases} g(f(x))=0 \\ g(1)=0 \\ g(2\sqrt{2})=0 \end{cases} \Rightarrow \begin{cases} f(x)=1=x\sqrt{x} \xrightarrow{()^2} x^3=1 \Rightarrow \boxed{x=1} \\ f(x)=2\sqrt{2}=x\sqrt{x} \xrightarrow{()^2} x^3=8 \Rightarrow \boxed{x=2} \end{cases}$ ✓

$\Rightarrow 2-1=1$ ✓

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