«دامنه‌های تحلیلی» (گروه A) ۱۶

۱- $f(\frac{\pi}{3}) \to \cot\frac{\pi}{\frac{3}{4}} \to \cot\frac{\pi}{12} \to \cot\frac{\pi}{6} \to \sqrt{3}$

$f(\sqrt{3}) \to \sqrt{3+1} \to (2)$ ✓ ۳

۲- الف) $g(\frac{\pi}{3}) \to 2\cos\frac{2\pi}{3} \to \frac{1}{2}$, $f(\frac{1}{2}) \to \sqrt{\frac{3(4)-12}{4}} \to \frac{\sqrt{3}}{2}$ ✓ ۲

ب) $g(\sqrt{2}) \to \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} \to \frac{\sqrt{2}+2}{1-2} \to -\sqrt{2}-2 \to f(-\sqrt{2}-2) \to [-\sqrt{2}-2] = -4$ ✓

۳- $f(\frac{\pi}{4}) \to \sin\frac{\pi}{4} \to \frac{\sqrt{2}}{2}$, $g(\frac{\sqrt{2}}{2}) \to \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} \to \frac{\sqrt{2}}{2}\sqrt{\frac{1}{2}} \to \sqrt{\frac{2}{4}\times\frac{1}{2}} \to \sqrt{\frac{1}{4}} = \frac{1}{2}$ ✓ ۲

۴- $fog(x) \to \{(4,5)(2,5)(6,12)(8,12)\}$ ✓ ۲

$gof(x) \to \varnothing$ ✓ $fof \to \varnothing$ ✓ $gog(x) \to \{(4,8)(2,6)(6,10)\}$ ✓

۵- برعکس حساب می‌کنیم ←

$(2,4) \in fog: 2 \to 3 \to a=4$

$(4,1) \in gof: 4 \xrightarrow{f(x)} 5 \xrightarrow{g(x)} 1$, $5 = b$

$\Rightarrow (4,5)$ ✓ ۳

۶- $f(f(x)) = 4x+3 \to f(x) = 2x+1$ یا $f(x) = -2x-3$

$g(\underbrace{2x+3=t}) \Rightarrow g(t) = 3(\frac{t-3}{2}) - 2 \to g(t) = \frac{3t-9}{2} - 2 \to g(x) = \frac{3x-13}{2}$ ۱

$\frac{t-3}{2} = x$, $\frac{3t-9-4}{2}$

$g(-1) = -8$

$g(x) = \frac{3x-9}{2} - 2$

$f(-1) \to -2+1 = -1 \to g(-1) \Rightarrow \frac{-3-13}{2} = -8$

۷- $D_{gof(x)} = \{x \in D_f \mid f(x) \in D_g\}$ ۵

$gof \to \frac{}{(\sqrt{x+|x|})^2 - 4\sqrt{x+|x|} + 3 - 3} \to \frac{}{(\sqrt{x+|x|}-2)^2 - 4 = 0} \to \sqrt{x+|x|} = 0 \to x \le 0$, $\sqrt{x+|x|} = 4$

$D_f = \mathbb{R}$, $D_g = \mathbb{R} - \{0,4\}$

$D_{gof} = (0,+\infty) - \{\frac{-1+\sqrt{65}}{2}\} \Leftarrow \frac{-1 \pm \sqrt{1+64x}}{2} \Leftarrow x+|x|-16=0$

$f(x) \neq 0,4 \to \begin{cases} \sqrt{x+|x|} \neq 0 \to x>0 \\ \sqrt{x+|x|} \neq 4 \to x \neq 8 \end{cases}$

$D_{gof} = (0,+\infty) - \{8\}$

۸- $\sqrt{1-x^2} + \sqrt{x} \to \sqrt{1-(\sqrt{1-x^2})^2} + \sqrt{\sqrt{1-x^2}} \to \sqrt{1-1+x^2} \to |x|$ ۲

$\leftarrow 1-x^2 \ge 0 \leftarrow \sqrt{1-x^2} \ge 0 \leftarrow |x| + \sqrt{\sqrt{1-x^2}}$

$-1 \quad 1$; $- \; + \; -$

$\to D_{(f+g)of} = [-1,1]$ ✓ ۱

۹- $f(\frac{3x+1}{x-2}) = 4x+5 \to tx - 2t = 3x+1 \to tx - 3x = 2t+1 \to x = \frac{2t+1}{t-3}$, $f(t) = \frac{8t+4}{t-3} + 5$

$f(x) = \frac{13x-11}{x-3}$

$f(x) = \frac{8x+4}{4x-12} + 5$

$f(x+\frac{1}{x}) = x^3 + \frac{1}{x^3} + 3x + \frac{3}{x} - 3x - \frac{3}{x} \Rightarrow f(x) = x^3 - \frac{1}{x^3} - 3x$, $(x+\frac{1}{x})^3$

$x+\frac{1}{x} = t$

$f(x) = x^3 - 3x$

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$f(x) = \sqrt{x^{۳}} \longrightarrow x \geq ۰$

$\sqrt{x^{۳}} = ۲\sqrt{۸} \Rightarrow x = ۲$

$\sqrt{x^{۳}} = ۱ \longrightarrow x = ۱$

$۲ - ۱ = ①$ ✓

(۲)