

$$f \circ f\left(\frac{2}{3}\right) \rightarrow \frac{2}{3} \times \frac{1}{4} \times \pi = \frac{\pi}{6} = 30^\circ \rightarrow 60 + 30 = 90^\circ$$

$$f(\sqrt{3}) \Rightarrow \sqrt{(\sqrt{3})^2 + 1} = 2$$

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الف) $2 \cos \frac{\pi}{3} = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{\sqrt{1 - \frac{1}{4}}}$

$$\Rightarrow \frac{\sqrt{3}}{\frac{1}{2}} = 2\sqrt{3}$$

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ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} \rightarrow \frac{\sqrt{2} + 2}{1 - 2} = \frac{\sqrt{2} + 2}{-1}$

$$\Rightarrow -(\sqrt{2} + 2)$$

$$f(-\sqrt{2} - 2) = [-\sqrt{2} - 2] \rightarrow [-\sqrt{2}] - 2 = -2 - 2 = -4$$

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$$f\left(\frac{\pi}{4}\right) \Rightarrow \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

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الف) $f \circ g(x) = \{(4, 5), (2, 5), (6, 2), (8, 2)\}$

ب) $g \circ f(x) = \emptyset$

ج) $f \circ f(x) = \emptyset$

د) $g \circ g(x) = \{(4, 1), (2, 4), (4, 0)\}$

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$$(2, 2) \in f \circ g \rightarrow \left. \begin{array}{l} 2 \in R_f \\ 2 \in D_g \\ a = 4 \end{array} \right\} \rightarrow (a, 2) \Rightarrow (4, 2) \Rightarrow a = 4$$

$$(4, 4)$$

$$(4, 1) \in g \circ f \rightarrow \left. \begin{array}{l} 1 \in R_g \\ 4 \in D_f \end{array} \right\} \rightarrow (4, 1) \rightarrow (b, 1) \Rightarrow b = 4$$

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$$f \circ f = a(an+b)+b \Rightarrow a^2x+ab+b = \varepsilon x + \gamma$$

$$a^2 = \varepsilon \Rightarrow a = \pm \sqrt{\varepsilon} \quad ab+b = \gamma \rightarrow b=1 \Rightarrow b = -\gamma$$

$$a = \sqrt{\varepsilon}, b = 1 \Rightarrow f(x) = \sqrt{\varepsilon}x + 1$$

$$a = -\sqrt{\varepsilon}, b = -\gamma \Rightarrow f(x) = -\sqrt{\varepsilon}x - \gamma \rightarrow f(-1) = -1$$

$$g(x) = cx + d \Rightarrow c(\sqrt{\varepsilon}x + 1) + d = \sqrt{\varepsilon}x - \gamma$$

$$\sqrt{\varepsilon}c + c + d = \sqrt{\varepsilon} - \gamma$$

$$\sqrt{\varepsilon}x + \gamma = -1$$

$$\sqrt{\varepsilon}x = -1 - \gamma \quad x = -\frac{1+\gamma}{\sqrt{\varepsilon}}$$

$$\text{if } x = -\frac{1+\gamma}{\sqrt{\varepsilon}} \Rightarrow g(-1) = -1$$

$$\begin{cases} \sqrt{\varepsilon}c + d = -\gamma \\ \sqrt{\varepsilon}c + c + d = \sqrt{\varepsilon} - \gamma \end{cases} \Rightarrow \frac{1}{\sqrt{\varepsilon}} + d = -\gamma \Rightarrow d = -\frac{1}{\sqrt{\varepsilon}} - \gamma \Rightarrow g(x) = \frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}}x - \frac{1}{\sqrt{\varepsilon}} - \gamma$$

$$\rightarrow g(-1) = \frac{-1}{\sqrt{\varepsilon}} - \gamma = -\frac{1}{\sqrt{\varepsilon}} - \gamma = g \circ f(-1)$$

$$Dg \circ f = \{x \in D_f : f(x) \in Dg\}$$

$$D_f = \mathbb{R}$$

$$x^2 - \varepsilon x = x(x - \varepsilon) \Rightarrow x \leq \varepsilon \Rightarrow Dg = \mathbb{R} - \{0, \varepsilon\} \rightarrow Dg \circ f = \{x \in \mathbb{R} : \sqrt{x+|x|} \neq 0, \varepsilon\}$$

$$x > 0, x \neq \varepsilon \Rightarrow Dg \circ f = (0, \infty) - \{\varepsilon\}$$

$$(F+g) \circ f = F(F(x)) + G(F(x))$$

$$Df_x \Rightarrow x \leq 1$$

$$\sqrt{1-x^2} \leq 1$$

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$$F(F(x)) \Rightarrow \sqrt{1-(\sqrt{1-x^2})^2} = \sqrt{x^2} = |x|$$

$$G(F(x)) = \sqrt{\sqrt{1-x^2}} = \sqrt{1-x^2}$$

$$(F+g) \circ f = |x| + \sqrt{1-x^2}$$

$$1-x^2 \geq 0 \quad -1 \leq x \leq 1$$

$$[-1, 1]$$

$$f\left(\frac{\gamma x+1}{x-\gamma}\right) \Rightarrow t = \frac{\gamma x+1}{x-\gamma} \quad t(x-\gamma) = \gamma x+1$$

(w)

$$tx - \gamma t = \gamma x + 1 \Rightarrow tx - \gamma x = \gamma t + 1$$

$$\Rightarrow x(t - \gamma) = \gamma t + 1 \Rightarrow x = \frac{\gamma t + 1}{t - \gamma}$$

$$F(t) = \varepsilon x + \omega \Rightarrow \frac{1 + \gamma t}{t - \gamma} + \omega$$

$$\Rightarrow \frac{1 + \gamma t - 1}{t - \gamma} \Rightarrow f(x) = \frac{1 + \gamma x - 1}{x - \gamma}$$

$$f\left(x + \frac{1}{x}\right) = x^{\gamma} + \frac{1}{x^{\gamma}}$$

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$$f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^{\gamma} - \gamma \left(x + \frac{1}{x}\right)$$

$$f(x) = x^{\gamma} - \gamma x$$

$$g \circ f = 0 \quad f(x) = 1 \rightarrow \frac{x\sqrt{x} - 1}{x} = 1$$

$$f(x) = x\sqrt{x} \rightarrow \frac{x\sqrt{x} - 1}{x} = \sqrt{x} - \frac{1}{x}$$

$$\sqrt{x} - \frac{1}{x} = 1$$