

در این سوال

A =

استفاده کنید

نام و نام خانوادگی = علی ربانی

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$f\left(\frac{1}{2}\right) = \cot \frac{\pi}{8} = \sqrt{2}$$

$$f(\sqrt{2}) = \sqrt{2^2 + 1} = \sqrt{5}$$

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$$f \circ f\left(\frac{1}{2}\right) = ?$$

$$\text{الف) } f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{4} - 1} \quad g(x) = \sqrt{\cos x}$$

$$f \circ g\left(\frac{\pi}{2}\right) = \sqrt{\cos \frac{\pi}{2}} = \sqrt{\frac{1}{2}} \quad f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{4} - 1} = \sqrt{\frac{3}{4}}$$

$$\text{ب) } f(x) = \sin x \quad g(x) = 2\sqrt{1-x^2}$$

$$g \circ f\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2} \quad g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$f(x) = \sin x$$

$$g(x) = 2\sqrt{1-x^2}$$

$$g \circ f\left(\frac{\pi}{2}\right) = g\left(f\left(\frac{\pi}{2}\right)\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2} \Rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{1}{2}$$

۳ -

$$f(x) = \{(1,0), (2,0), (3,1), (4,1)\} \quad g(x) = \{(1,1), (2,1), (3,2), (4,2)\}$$

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$$\text{الف) } f \circ g(x) = \{(1,0), (2,0), (3,1), (4,1)\} \quad \text{ب) } g \circ f(x) = \emptyset$$

$$\begin{array}{l} 1,0 \xrightarrow{f} 0 \\ 2,0 \xrightarrow{f} 0 \\ 3,1 \xrightarrow{f} 1 \\ 4,1 \xrightarrow{f} 1 \end{array}$$

$$\begin{array}{l} 1,1 \xrightarrow{g} x \\ 2,1 \xrightarrow{g} x \\ 3,2 \xrightarrow{g} x \\ 4,2 \xrightarrow{g} x \end{array}$$

$$\text{ج) } f \circ f(x) = \emptyset$$

$$\text{د) } g \circ g(x)$$

$$= \{(1,1), (2,1), (3,2)\}$$

$$\begin{array}{l} 1,0 \xrightarrow{f} x \\ 2,0 \xrightarrow{f} x \\ 3,1 \xrightarrow{f} x \\ 4,1 \xrightarrow{f} x \end{array}$$

$$\begin{array}{l} 1,1 \xrightarrow{g} 1 \\ 2,1 \xrightarrow{g} 2 \\ 3,2 \xrightarrow{g} 3 \\ 4,2 \xrightarrow{g} x \end{array}$$

$$F = \{(1,1), (2,1), (3,2), (4,2)\} \quad g = \{(1,1), (2,1), (3,2), (4,2)\}$$

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$$(1,1) \in f \circ g \quad (1,1) \in g \circ f \quad (a,b) = (1,1)$$

$$g(a) = 1 \xrightarrow{f} 1 \Rightarrow a = 1 \quad f(1) = 1 \xrightarrow{g} 1 \Rightarrow b = 1$$

۶ -

$$f(f(x)) = x + 2$$

$$g(x, y) = x - y$$

$$a(ax+b)+b = x+2$$

$$g \circ f(-1) = f(-1) = -1 + 2 = 1$$

$$a^2x + ab + b = x + 2$$

$$g(-1) = -1$$

$$a^2 = 1 \Rightarrow a = \pm 1 \Rightarrow f(x) = x + 1$$

$$(x+y) = -1 \Rightarrow g(x+y, x) = (x+y) - x = y = -1 \Rightarrow x = -2$$

$$f(x) = \sqrt{x+|x|} \quad g(x) = \frac{1}{x^2 - x_2} \quad D_f = \mathbb{R} \quad D_g = \mathbb{R} - \{0, x_2\} \quad -V$$

$$D_{g \circ f} = \{x \in D_f, f(x) \in D_g\} = \{x \in \mathbb{R}, \sqrt{x+|x|} \neq 0, x_2 \Rightarrow x \neq \mathbb{R}, \wedge$$

$$D_{g \circ f} = [0, +\infty) - \{x_2\}$$

$x+|x| \begin{cases} \rightarrow x & x \geq 0 \\ \rightarrow 0 & x < 0 \end{cases} \quad \sqrt{x+|x|} \neq x_2 \Rightarrow x \neq \mathbb{R}$

$$f+g = \sqrt{1-x^2} + \sqrt{x} \Rightarrow D_{f+g} = [-1, 1] \cap [0, +\infty) = [0, 1] \quad \wedge$$

$$D_{(f+g) \circ f} = x \in D_f, f(x) \in D_{(f+g)}$$

$$\sqrt{1-x^2} \in [0, 1] \Rightarrow (0 \leq \sqrt{1-x^2} \leq 1) \quad \begin{cases} 0 \leq 1-x^2 \leq 1 \\ -1 \leq -x^2 \leq 0 \\ 1 \geq x^2 \geq 0 \end{cases}$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$\Leftarrow$

$$-) f(x + \frac{1}{x}) = x^2 + \frac{1}{x^2}$$

$$f(x + \frac{1}{x}) = (x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + \left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} + x + \frac{1}{x}$$

$$\Rightarrow f(x + \frac{1}{x}) = (x + \frac{1}{x})^2 - \left(x + \frac{1}{x}\right) \Rightarrow f(x) = x^2 - \frac{1}{x}$$

$$\text{و) } f\left(\frac{x+1}{x-1}\right) = x^2 + \Delta \Rightarrow \frac{x+1}{x-1} = t \Rightarrow f(t) = x^2 + \Delta \Rightarrow f = x^2 \left(-\frac{x-1}{x+1}\right) + \Delta$$

$$x+1 = t(x-1) \quad (x-1)x = tx-1 \Rightarrow x = \frac{-tx-1}{x-t} = \frac{-tx-1}{x-t} \quad \Delta = \frac{-tx-1}{x-t}$$

$$f(x) = \frac{-1(x^2+1)}{x-1} \quad \& \quad \frac{1(x-1)}{x-1}$$

$$\Leftarrow = \frac{-1(x^2+1)}{x-t}$$

$$g \circ f(x) \quad f(x) = x\sqrt{x}$$

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$$g(1) = 0$$

$$g(\sqrt{x}) = 0$$

$$f(x) = 1 \Rightarrow x = 1$$

$$f(x) = \sqrt{x} \Rightarrow x = 1$$

$$\{1\} = \{1\}$$