

در این سوال

A = ...

با استفاده از ...

نام و نام خانوادگی = علی ربانی

$$f(x) = \begin{cases} \cot \frac{\pi x}{2} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$f\left(\frac{2}{2}\right) = \cot \frac{\pi}{2} = \sqrt{2}$$

$$f(\sqrt{2}) = \sqrt{2+1} = \sqrt{3}$$

$$f \circ f\left(\frac{2}{2}\right) = ?$$

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{4} + 1} = \sqrt{\frac{5}{4}} \quad g(x) = \sqrt{2} \cos x$$

$$f \circ g\left(\frac{\pi}{2}\right) = \sqrt{2} \cos \frac{\pi}{2} = \frac{1}{2} \quad f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{4} + 1} = \sqrt{\frac{5}{4}}$$

$$f(x) = \sin x \quad g(x) = 2\sqrt{1-x^2} \quad f \circ g(\sqrt{2}) \rightarrow g(\sqrt{2}) = -\sqrt{2} - 2 \quad f(-\sqrt{2}-2) = -4$$

$$g \circ f\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2} \quad g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$f(x) = \sin x \quad g(x) = 2\sqrt{1-x^2} \quad g \circ f\left(\frac{\pi}{2}\right) = g\left(f\left(\frac{\pi}{2}\right)\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2} \Rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{1}{2}$$

$$f(x) = \{(1,0), (2,0), (3,1), (4,1)\} \quad g(x) = \{(1,0), (2,1), (3,1), (4,1)\}$$

$$f \circ g(x) = \{(1,0), (2,0), (3,1), (4,1)\} \quad g \circ f(x) = \emptyset$$

$$\begin{array}{ccc} 1 & \xrightarrow{f} & 0 \\ 2 & \xrightarrow{f} & 0 \\ 3 & \xrightarrow{f} & 1 \\ 4 & \xrightarrow{f} & 1 \end{array}$$

$$\begin{array}{ccc} 1 & \xrightarrow{g} & 0 \\ 2 & \xrightarrow{g} & 1 \\ 3 & \xrightarrow{g} & 1 \\ 4 & \xrightarrow{g} & 1 \end{array}$$

$$f \circ f(x) = \emptyset$$

$$g \circ g(x) = \{(1,0), (2,1), (3,1), (4,1)\}$$

$$\begin{array}{ccc} 1 & \xrightarrow{f} & 0 \\ 2 & \xrightarrow{f} & 0 \\ 3 & \xrightarrow{f} & 1 \\ 4 & \xrightarrow{f} & 1 \end{array}$$

$$\begin{array}{ccc} 1 & \xrightarrow{g} & 0 \\ 2 & \xrightarrow{g} & 1 \\ 3 & \xrightarrow{g} & 1 \\ 4 & \xrightarrow{g} & 1 \end{array}$$

$$f = \{(1,0), (2,1), (3,0), (4,1)\}$$

$$g = \{(1,0), (2,1), (3,1), (4,1)\}$$

$$(1,0) \in f \circ g$$

$$(1,0) \in g \circ f$$

$$(a,b) = (1,0)$$

$$g(a) = 0 \Rightarrow a = 1$$

$$f(a) = 0 \Rightarrow a = 1$$

$$f(f(x)) = x+2$$

$$g(x+2) = x-2$$

$$f \circ g = -1$$

$$a(a+b)+b = x+2$$

$$g \circ f(-1) = f(-1) = -1+2 = 1$$

$$a^2x + ab + b = x+2$$

$$g(-1) = -1$$

$$a^2 = 1 \Rightarrow a = 1 \Rightarrow f(x) = x+1$$

$$(x+2) = -1 \Rightarrow g(x+2) = (x+2)-2 = x$$

$$\Rightarrow x+2 = x \Rightarrow b = -2 \Rightarrow f(x) = x+2$$

$$\Rightarrow x = -2$$

$$f(x) = \sqrt{x+|x|} \quad g(x) = \frac{1}{x^2 - x_2} \quad D_f = \mathbb{R} \quad D_g = \mathbb{R} - \{0, x_2\} \quad -V$$

$$D_{g \circ f} = \{x \in D_f, f(x) \in D_g\} = \{x \in \mathbb{R}, \sqrt{x+|x|} \neq 0, x_2 \Rightarrow x \neq \mathbb{R}_0^- \wedge$$

$$D_{g \circ f} = \boxed{[1, +\infty) - \{x_2\}} \quad \begin{matrix} x+|x| \begin{cases} \rightarrow x & x \geq 0 \\ \rightarrow 0 & x < 0 \end{cases} \end{matrix} \quad \sqrt{x+|x|} \neq x_2 \Rightarrow x \neq x_2 \quad (1, \infty)$$

$$f+g = \sqrt{1-x^2} + \sqrt{x} \Rightarrow D_{f+g} = [-1, 1] \cap [0, +\infty) = [0, 1] \quad \wedge$$

$$D_{(f+g) \circ f} = x \in D_f, f(x) \in D_{(f+g)}$$

$$\sqrt{1-x^2} \in [0, 1] \Rightarrow (0 \leq \sqrt{1-x^2} \leq 1) \quad \begin{matrix} 0 \leq 1-x^2 \leq 1 \\ -1 \leq -x^2 \leq 0 \\ 1 \geq x^2 \geq 0 \end{matrix}$$

$$D_{(f+g) \circ f} = \boxed{[-1, 1]} \quad \checkmark \quad (Y)$$

$$-) f(x + \frac{1}{x}) = x^2 + \frac{1}{x^2}$$

$$f(x + \frac{1}{x}) = (x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + \left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} + x + \frac{1}{x}$$

$$\Rightarrow f(x + \frac{1}{x}) = (x + \frac{1}{x})^2 - \left(x + \frac{1}{x}\right) \Rightarrow f(x) = x^2 - \frac{1}{x^2} \quad \checkmark$$

$$a) f\left(\frac{x+1}{x-1}\right) = x^2 + \Delta \Rightarrow \frac{x+1}{x-1} = t \Rightarrow f(t) = t^2 + \Delta \Rightarrow t = \left(\frac{-t-1}{t-1}\right) + \Delta$$

$$x+1 = t(x-1) \quad (x-1)t = x-1 \Rightarrow x = \frac{-t-1}{t-1} = \frac{-1t-1}{t-1} \quad \Delta = \frac{-1t-1}{t-1} + \Delta$$

$$f(x) = \frac{-1x^2+1}{x-1} \quad \text{or} \quad \frac{1x^2-1}{x-1} \quad \checkmark \quad (Y) \quad \Leftarrow \quad = \frac{-1x^2+1}{x-1}$$

$$g \circ f(x)$$

$$f(x) = x\sqrt{x}$$

$$(1) - 10$$

$$g(1) = 0$$

$$g(\sqrt{x}) = 0$$

$$f(x) = 1 \Rightarrow x = 1$$

$$f(x) = \sqrt{x} \Rightarrow x = 1$$

$$\checkmark \quad (1) = 1$$

$$(Y)$$