

$$f(x) = \sqrt{1-x^2} \rightarrow 1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow D_f = [-1, 1]$$

$$g = \{(-1, 1), (0, 4), (2, 0), (1, 2)\} \Rightarrow D_g = \{-1, 0, 2, 1\}$$

$$D_f \cap D_g \Rightarrow \{-1, 0, 1\}$$

$$\begin{cases} f(1) = 0, g(1) = 2 \Rightarrow (2g - 3f)(1) = 4 \\ f(0) = 1, g(0) = 4 \Rightarrow (2g - 3f)(0) = 8 \\ f(-1) = 0, g(-1) = 1 \Rightarrow (2g - 3f)(-1) = 2 \end{cases}$$

$$\Rightarrow 2g - 3f = \{(1, 4), (0, 8), (-1, 2)\} \Rightarrow \text{مجموعه مقادیر} = \{2, 4, 8\}$$

$$f(x) = 2x - 1 \quad \begin{cases} \text{تابع خطی} \\ D_f = [2, +\infty) \end{cases} \quad \begin{cases} R_f = [2, +\infty) \\ \text{بازتاب + است} \end{cases}$$

$$g(x) = \frac{1}{x} + 3 \quad \begin{cases} \text{تابع و یک تابع خطی} \\ D_g = (-\infty, 3] \end{cases} \quad \begin{cases} R_g = (-\infty, 3] \\ \text{بازتاب + است} \end{cases}$$

$$\Rightarrow (-\infty, 3] \cup [2, +\infty)$$

$$y = \frac{-x^2}{2} + x + 3$$

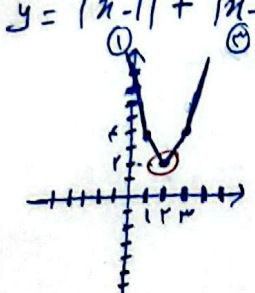
$$(a, b) \text{ (I)}$$

$$\frac{-x^2}{2} + x + 3 > \frac{3}{2} \Rightarrow \frac{-x^2}{2} + x + \frac{3}{2} > 0$$

$$\Rightarrow -x^2 + 2x + 3 > 0 \Rightarrow (x+1)(-x+3) > 0 \Rightarrow \frac{-1}{-1} + \frac{3}{1} = 2$$

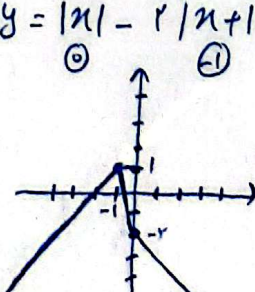
$$(II) \Rightarrow a = -1, b = 3 \Rightarrow \sqrt{b-a} = \sqrt{4} = 2$$

$$(III) \Rightarrow \boxed{(-1, 3)}$$

$$y = |x-1| + |x-2| + |2x-1|$$


1	2	3
$-x+1-x+2$	$x-1-x+2$	$x-1-x+2$
$-2x+3$	$-2x+2$	$2x-1$
$= -4x+3$	$= -2x+3$	$= 2x-1$

$$\Rightarrow \text{کمترین مقدار} = 2$$

$$y = |x| - 2|x+1|$$


-1	0
$-x+2x+2$	$x-2x-2$
$= x+2$	$= -x-2$

$$\Rightarrow R_y = (-\infty, 1]$$

$$y = \frac{x^2 + dx + m}{x+1} \Rightarrow xy + y = x^2 + dx + m \Rightarrow x^2 + (d-y)x + (m-y) = 0$$

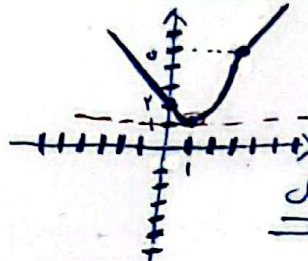
$$\Rightarrow x = \frac{y-d \pm \sqrt{(y-d)^2 - 4(m-y)}}{2} \Rightarrow y = 2y + 2d - 4m \Rightarrow 0 = 0$$

$$\Rightarrow 3d - 100 + 4m \leq 0 \Rightarrow 13m - 4 \leq 0$$

$$\Rightarrow m \in \mathbb{N} \Rightarrow m = \{1, 2, 3, 4\}$$

(الف) مقدار طبیعی m وجود دارد

$$f(x) = \begin{cases} x+2 & ; x \geq 2 \\ x^2 - rx + r & ; 0 \leq x < 2 \\ |x| + r & ; x < 0 \end{cases}$$



$$y = x^2 - rx + r \Rightarrow x_s = -\frac{-r}{2} = -\frac{r}{2} = 1$$

$$y_s = 1 - r + r = 1$$

$$R_f = [1, +\infty)$$

$$f(x) = \begin{cases} x^2 + rx + r & ; x \leq 0 \\ [rx] - rx & ; x > 0 \end{cases} \Rightarrow y = [a] - a = x_s = (-1, 0]$$

$$y = x^2 + rx + r \Rightarrow x_s = -\frac{r}{2} = -\frac{r}{2} = -1 \Rightarrow y_1 = x^2 + rx + r \Big|_{x=-1} = 1 - r + r = 1$$

$$\Rightarrow R_{y_1} = [-b, +\infty)$$

$$R_{y_1} \cup R_{y_2} \Rightarrow R_f = [-1, +\infty)$$

$$y = a + 1 - \sqrt{rx + r} \Rightarrow rx + r \geq 0 \Rightarrow x \geq -\frac{r}{r} \Rightarrow D_y = [-\frac{r}{r}, +\infty)$$

$$D_y = [b, +\infty) \Rightarrow b = -\frac{r}{r}$$

$$R_y = (-\infty, d] \Rightarrow d = a + 1 - \sqrt{rx + r} \Big|_{x=-\frac{r}{r}} = a + 1 - \sqrt{r(-\frac{r}{r}) + r} = a + 1 - \sqrt{-r + r} = a + 1$$

$$\Rightarrow R_y = (-\infty, a + 1]$$

$$f(x) = r\sqrt{x+1} + r\sqrt{1-x} \Rightarrow D_f = [-1, 1] = D_g = D_h$$

$$f(x) + g(x) = r\sqrt{r+1}\sqrt{1-x} \Rightarrow g(x) = r\sqrt{r+1}\sqrt{1-x} - f(x) = r\sqrt{r+1}\sqrt{1-x} - r\sqrt{x+1} - r\sqrt{1-x}$$

$$y = \frac{f(x)}{g} - \frac{h(x)}{r} = \frac{r\sqrt{x+1} + r\sqrt{1-x}}{r} - \frac{r\sqrt{r+1}\sqrt{1-x} - r\sqrt{x+1} - r\sqrt{1-x}}{r} = \frac{\sqrt{x+1} + \sqrt{1-x}}{1} - \frac{\sqrt{r+1}\sqrt{1-x} - \sqrt{x+1} - \sqrt{1-x}}{1}$$

$$= \sqrt{x+1} + \sqrt{1-x} - \sqrt{r+1}\sqrt{1-x} + \sqrt{x+1} + \sqrt{1-x} = 2\sqrt{x+1} + 2\sqrt{1-x} - \sqrt{r+1}\sqrt{1-x}$$

$$= \sqrt{x+1} + \sqrt{1-x} - \sqrt{1-x} = \sqrt{x+1} \Rightarrow R_y = [0, \sqrt{r}]$$