آردین ملکیان یازدهم A ۱۹,۵

۱- (۲)

$f(x)=\sqrt{1-x^2} \rightarrow 1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow D_f=[-1,1]$

$g=\{(-1,1),(0,4),(2,0),(1,2)\} \Rightarrow D_g=\{-1,0,2,1\}$

$D_f \cap D_g \Rightarrow \{-1,0,1\}$

$f(1)=0 \,,\, g(1)=2 \Rightarrow (2g-3f)(1)=4$

$f(0)=1 \,,\, g(0)=4 \Rightarrow (2g-3f)(0)=5$

$f(-1)=0 \,,\, g(-1)=1 \Rightarrow (2g-3f)(-1)=2$

$\Rightarrow 2g-3f=\{(1,4),(0,5),(-1,2)\}$

$\Rightarrow$ مجموع اعضای برد $=4+5+2=11$ ✓

۲- (۲)

$f(x)=2x-1 \,,\, D_f=[3,+\infty)$ $\xrightarrow[\text{شیب +}]{\text{اکیداً صعودی}}$ $R_f=[5,+\infty)$ $\quad (2\times3)-1$

$g(x)=\frac{1}{3}x+3 \,,\, D_g=(-\infty,3)$ $\xrightarrow[\text{شیب +}]{\text{اکیداً صعودی}}$ $R_g=(-\infty,4]$

$\xRightarrow{R_f \cup R_g} (-\infty,4] \cup [5,+\infty)$ ✓

۳- (۲)

$y=-\frac{x^2}{2}+x+3$

$-\frac{x^2}{2}+x+3 > \frac{3}{2} \Rightarrow -\frac{x^2}{2}+x+\frac{3}{2} > 0$

$\Rightarrow (x+1)(-x+3) > 0 \Rightarrow$

	-1		3	
−		+		−

(a,b) → $(-1,3)$

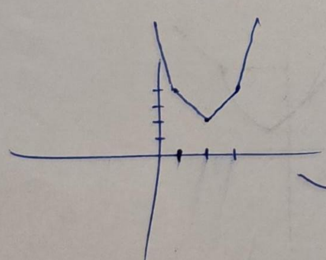
$a=-1 \,,\, b=3 \Rightarrow \sqrt{b-a}=\sqrt{4}=2$ ✓

۴- (۲)

$y=|x-1|+|x-3|+|2x-4|$

① ③ ②

$x<1$	$1<x<2$	$2<x<3$	$x>3$
$-x+1-x+3-2x+4=-4x+8$ ④	$x-1-x+3-2x+4=-2x+6$ ③	$x-1-x+3+2x-4=2x-2$ ④	$x-1+x-3+2x-4=4x-8$

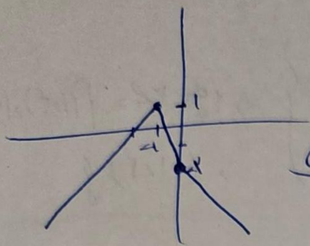


$\min=2$ ✓

۵- (۲)

$y = |x| - 2|x+1|$ (0) (-1)

	-1	0
$-x+2x+2 = x+2$	$-x-2x-2 = -3x-2$	$x-2x-2 = -x-2$
(1)		(-2)



شکل → $R_y = (-\infty, 1]$ ✓

۶- (۱٫۵)

$y = \frac{x^2+5x+m}{x+1} \Rightarrow xy + y = x^2 + 5x + m \Rightarrow x^2 + (5-y)x + m - y = 0$

$\Rightarrow x = \frac{y - 5 \pm \sqrt{y^2 - 10y + 25 + 4y - 4m}}{2} \rightarrow y^2 - 6y + 25 - 4m \geq 0$

$\Rightarrow 36 - 100 + 16m \leq 0 \Rightarrow 16m - 64 \leq 0 \Rightarrow m \leq 4$, $m \in N$ } $m = \{1, 2, 3, 4\}$

اگه $m = 4$ باشه ۱۳ زیر حذف میشه و دیگه IR نیست پس $m = \{1, 2, 3\}$

۴ مقدار طبیعی برای m

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۱۸ (۲)

$f(x) = \begin{cases} x^2 + 4x + 3 ; & x \leq 0 \\ [2x] - 2x ; & x > 0 \end{cases}$ $2x = a \Rightarrow y_2 = [a] - a \Rightarrow R_{y_2} = (-1, 0]$

$y_1 = x^2 + 4x + 3 \Rightarrow x_s = \frac{-b}{2a} = -2$ $\Rightarrow y_1 = x^2 + 4x + 3$ { $a > 0$, ext $| {-2 \atop -1}$ } $\Rightarrow R_{g_1} = [\ldots, +\infty)$

$y_s = 4 - 8 + 3 = -1$

$R_{y_1} \cup R_{y_2} \rightarrow R_f = [-1, +\infty)$ ✓

جابجا

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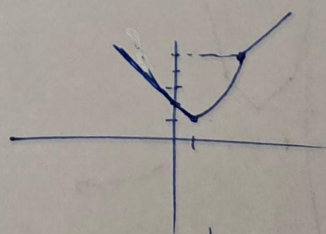
۱۷ (۲)

$f(x) = \begin{cases} \ldots x + 2 ; & x \geq 3 \overset{x=3}{\Rightarrow} y = 5 \\ x^2 - 2x + 2 ; & 0 \leq x < 3 \overset{x=0}{\Rightarrow} y = 2 \overset{x=3}{\Rightarrow} y = 5 \\ |x| + 2 & x < 0 \overset{x=0}{\Rightarrow} y = 2 \end{cases}$

$y = x^2 - 2x + 2 \Rightarrow x_s = -\frac{b}{2a} = -\frac{-2}{2} = 1$

$y_s = 1$



↓

$R_f = [1, +\infty)$ ✓

$y = a+1 - \sqrt{2x+3} \Rightarrow 2x+3 \geq 0 \Rightarrow x \geq -\frac{3}{2} \Rightarrow D_y = [-\frac{3}{2}, +\infty)$ (۲) -۹

$D_g = [b, +\infty)$ (I)

$R_g = (-\infty, d)$ (II)

$\Rightarrow b = -\frac{3}{2}$ ✓ $\quad ab = -4$ ✓ $\quad a = 4$ ✓

$\Rightarrow -\infty < a+1 - \sqrt{2x+3} \leq a+1 \Rightarrow R_g = (-\infty, a+1)$

$\downarrow$

d

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(۲) ۱۰

$f(x) = 2\sqrt{x+1} + 4\sqrt{1-x} \overset{I \cap II}{\Rightarrow} D_f = [-1,1] = D_g = D_g$

$f(x) + g(x) = 2\sqrt{2+2\sqrt{1-x^2}} \Rightarrow g(x) = 2\sqrt{2+2\sqrt{1-x^2}} - f(x) = 2\sqrt{2+2\sqrt{1-x^2}} - 2\sqrt{x+1} + 4\sqrt{1-x}$

$y = \frac{f(x)}{4} - \frac{g(x)}{2} = \frac{2\sqrt{x+1} + 4\sqrt{1-x}}{4} - \frac{2\sqrt{2+2\sqrt{1-x^2}} - 2\sqrt{x+1} - 4\sqrt{1-x}}{2}$

$= \frac{\sqrt{x+1}}{2} + \frac{2\sqrt{x+1}}{2} + \sqrt{1-x} + 2\sqrt{1-x} - \sqrt{2+2\sqrt{1-x^2}} = \sqrt{x+1} + 2\sqrt{1-x} - \sqrt{(\sqrt{1-x}+\sqrt{1+x})^2}$

$= \sqrt{x+1} + 2\sqrt{1-x} - |\sqrt{1-x} + \sqrt{1+x}| = \sqrt{x+1} + 2\sqrt{1-x} - \sqrt{1-x} - \sqrt{1+x} = \sqrt{1-x}$ ✓

$D_g = [-1,1] \Rightarrow R_g = [0, \sqrt{2}]$ ✓