رادین ملکیان یازدهم A

۱) الف)

$y = x^3 - 3x^2 + 3x \longrightarrow R_f = \mathbb{R}$

$y = x^3 - 3x^2 + 3x - 1 + 1 = (x-1)^3 + 1 \Rightarrow y - 1 = (x-1)^3 \Rightarrow x = \sqrt[3]{y-1} + 1$

$\Rightarrow R_f = \mathbb{R}$

ب)

$y = \frac{1}{x^2 - 2x} = \frac{1}{(x-1)^2 - 1} \Rightarrow (x-1)^2 - 1 = \frac{1}{y} \Rightarrow (x-1)^2 = \frac{1}{y} + 1$

$x - 1 = \pm\sqrt{\frac{1}{y} + 1} \Rightarrow x = \pm\sqrt{\frac{1}{y}+1} + 1 \Rightarrow \frac{1}{y} + 1 \geq 0 \Rightarrow \frac{1}{y} \geq -1 \Rightarrow y > 0$

$y \leq -1$

$y \neq 0$

$\Rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

---

۲) الف)

$y = x^2 - 4x + 10$ $\begin{cases} a > 0 \quad \text{min دا} \\ -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2 \Rightarrow (2)^2 - 4(2) + 10 = 6 \end{cases} \Rightarrow R_f = [6, +\infty)$

ب)

$y = -x^2 + 6x + 3 \Rightarrow \begin{cases} a < 0 \quad \text{max دا} \\ -\frac{b}{2a} = -\frac{(-6)}{2(-1)} = 3 \Rightarrow -(3)^2 + 6(3) + 3 = 12 \end{cases}$

$\Rightarrow R_f = (-\infty, 12]$

ج)

$y = \sqrt{x^2 - 4x - 3}$ $\begin{cases} a > 0 \quad \text{min دا} \\ -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2 \end{cases} \Rightarrow y = \sqrt{(2)^2 - (4 \times 2) - 3} = \sqrt{-7}$ ✗

$\Rightarrow R_f = [0, +\infty)$

د)

$y = \sqrt{-x^2 + 6x}$ $\begin{cases} a < 0 \quad \text{max دا} \\ -\frac{b}{2a} = -\frac{(6)}{2(-1)} = 3 \end{cases} \Rightarrow y = \sqrt{6(3) - 3^2} = 3$

$\Rightarrow R_f = \sqrt{(-\infty, 9]} = [0, 3]$

---

۳)

الف) $y = x^3 - 5x^2 + 2x + 1 \Rightarrow R_f = \mathbb{R}$

ب) $y = x^7 - 6x^4 + 3x + 2 \Rightarrow R_f = \mathbb{R}$

ج) $y = \sqrt{x^7 - 6x^2 + 5x + 1} \Rightarrow R_f = [0, +\infty)$

د) $y = (x^5 - 4x^2 + 3x + 1)^2 \Rightarrow R_f = [0, +\infty)$

۴)

الف) $y = \dfrac{3x+1}{x-2} \Rightarrow R_f = \mathbb{R} - \{\frac{3}{1}\}$

ب) $y = \dfrac{2x+1}{x+1} \Rightarrow R_f = \mathbb{R} - \{2\}$

---

۵)

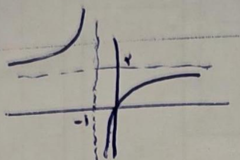
الف) $y = \sqrt{\dfrac{2x+4}{x+1}} \Rightarrow R_f = \sqrt{\mathbb{R} - \{2\}} = [0, +\infty) - \{\sqrt{2}\}$

ب) $y = \sqrt{\dfrac{3x+1}{2-x}} \Rightarrow R_f = \sqrt{\mathbb{R} - \{-3\}} = [0, +\infty)$

---

۶)

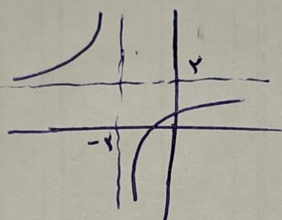
الف) $y = \dfrac{2x-3}{x+1}$



مجانب افقی $= \dfrac{a}{c} = 2$

ریشه مخرج $x = -1$ قائم

ب) $y = \dfrac{2x-1}{x+2}$



مجانب افقی $= \dfrac{a}{c} = 2$

قائم $x = -2$

---

۷)

الف) $y = \sin + \dfrac{1}{\sin}$ $\qquad R_f = (-\infty, -2] \cup [2, +\infty)$

$\sin > 0 \Rightarrow R_f = [2, +\infty)$

$\sin < 0 \Rightarrow R_f = (-\infty, -2]$

ب) $y = \dfrac{x^6+1}{x^3} \Rightarrow x^3 + \dfrac{1}{x^3} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \dfrac{\sqrt[3]{x^2+1}}{\sqrt[3]{x}} = \sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \dfrac{1}{\sqrt{x}} \Rightarrow R_f = [2, +\infty)$

۱۸)

الف) $y = x^۲ + \frac{۱}{x^۲+۳} = x^۲ + \frac{۱}{x^۲+۳} + ۳ - ۳ = x^۲+۳ + \frac{۱}{x^۲+۳} - ۳$

$x^۲+۳ \xrightarrow{a>0} min \Rightarrow -\frac{b}{۲a} = \frac{-0}{۲(۱)} = 0 \Rightarrow y_{min} = ۳ \Rightarrow ۳ + \frac{۱}{۳} - ۳ = \frac{۱}{۳}$

$R_f = [\frac{۱}{۳}, +\infty)$

ب) $y = \frac{x^۲+۵}{\sqrt{x^۲+۴}} = \frac{x^۲+۴+۱}{\sqrt{x^۲+۴}} = \sqrt{x^۲+۴} + \frac{۱}{\sqrt{x^۲+۴}} \Rightarrow x^۲+۴ \Rightarrow x_{min} = -\frac{b}{۲a} = 0$

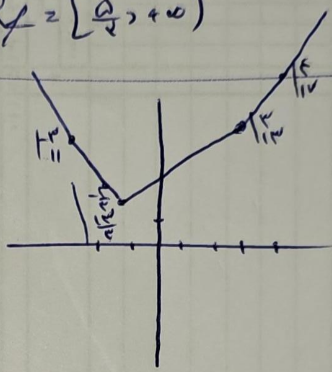
$y_{min} = ۴$

$\Rightarrow y = \sqrt{۴} + \frac{۱}{\sqrt{۴}} = ۲ + \frac{۱}{۲} = \frac{۵}{۲} \Rightarrow R_f = [\frac{۵}{۲}, +\infty)$

۱۹

$y = |x-۳| + |۳x+۴|$

$\begin{cases} x>۳ : x-۳+۳x+۴ = ۴x+۱ \\ -\frac{۴}{۳} \leq x \leq ۳ : ۳-x+۳x+۴ = ۲x+۷ \\ x < -\frac{۴}{۳} : ۳-x-۳x-۴ = -۴x-۱ \end{cases}$



---

۱۰)

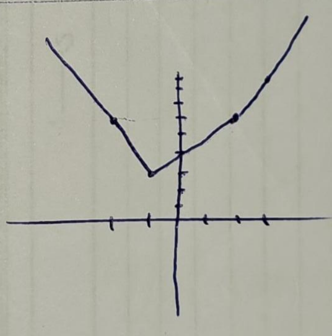
الف) $y = |x-۲| + |۲x+۲|$

$\xRightarrow{x=۲} y = |۲-۲| + |۲(۲)+۲| = ۶$

$\xRightarrow{x=-۱} y = |(-۱)-۲| + |۲(-۱)+۲| = ۳$

$\xRightarrow{x=۳} y = |۳-۲| + |۲(۳)+۲| = ۹$

$\xRightarrow{x=-۲} y = |(-۲)-۲| + |۲(-۲)+۲| = ۶$



$R_f = [۳, +\infty)$

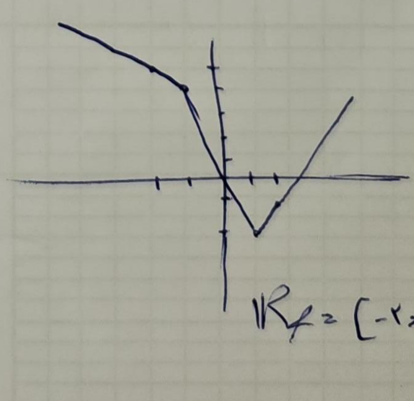
ب) $y = |۲x-۲| - |x+۱|$

$\xRightarrow{x=۱} y = |۲(۱)-۲| - |۱+۱| = -۲$

$\xRightarrow{x=-۱} y = |۲(-۱)-۲| - |(-۱)+۱| = ۴$

$\xRightarrow{x=۲} y = |۲(۲)-۲| - (۲+۱) = -۱$

$\xRightarrow{x=-۲} y = |۲(-۲)-۲| - |(-۲)+۱| = ۵$



$R_f = [-۲, +\infty)$