۱۹٫۵ پاردین تکلیان یازدهم A

۱) الف) ۲

$y = x^3 - 3x^2 + 3x \longrightarrow R_f = \mathbb{R}$

$y = x^3 - 3x^2 + 3x - 1 + 1 = (x-1)^3 + 1 \Rightarrow y - 1 = (x-1)^3 \Rightarrow x = \sqrt[3]{y-1} + 1$

$\Rightarrow R_f = \mathbb{R}$ ✓

ب)

$y = \frac{1}{x^2 - 2x} = \frac{1}{(x-1)^2 - 1} \Rightarrow (x-1)^2 - 1 = \frac{1}{y} \Rightarrow (x-1)^2 = \frac{1}{y} + 1$

$x - 1 = \pm\sqrt{\frac{1}{y} + 1} \Rightarrow x = \pm\sqrt{\frac{1}{y}+1} + 1 \Rightarrow \frac{1}{y} + 1 \geq 0 \Rightarrow \frac{1}{y} \geq -1 \Rightarrow y > 0$

$y \leq -1$

$y \neq 0$

$\Rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$ ✓

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۲) الف) ۲

$y = x^2 - 6x + 10$ $\begin{cases} a > 0 & \text{min دارد} \\ -\frac{b}{2a} = -\frac{(-6)}{2(1)} = 3 \Rightarrow (3)^2 - 6(3) + 10 = 1 \end{cases}$ $\Rightarrow R_f = [1, +\infty)$ ✓

ب)

$y = -x^2 + 6x + 3 \Rightarrow \begin{cases} a < 0 & \text{max دارد} \\ -\frac{b}{2a} = -\frac{(-6)}{2(-1)} = 3 \Rightarrow -(3)^2 + 6(3) + 3 = 12 \end{cases}$

$\Rightarrow R_f = (-\infty, 12]$ ✓

ج)

$y = \sqrt{x^2 - 4x - 3}$ $\begin{cases} a > 0 & \text{min دارد} \\ -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2 \end{cases}$ $\Rightarrow y = \sqrt{(2)^2 - (4 \times 2) - 3} = \sqrt{-7}$ ✗

$\Rightarrow R_f = [0, +\infty)$ ✓

د)

$y = \sqrt{-x^2 + 6x}$ $\begin{cases} a < 0 & \text{max دارد} \\ -\frac{b}{2a} = -\frac{(6)}{2(-1)} = 3 \end{cases}$ $\Rightarrow y = \sqrt{6(3) - 3^2} = 3$

$\Rightarrow R_f = \sqrt{(-\infty, 9]} = [0, 3]$ ✓

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۳) ۲

الف) $y = x^3 - 5x^2 + 2x + 1 \Rightarrow R_f = R$ ✓

ب) $y = x^7 - 9x^4 + 3x + 2 \Rightarrow R_f = R$ ✓

ج) $y = \sqrt{x^7 - 9x^2 + 5x + 1} \Rightarrow R_f = [0, +\infty)$ ✓

د) $y = (x^5 - 4x^2 + 3x + 1)^2 \Rightarrow R_f = [0, +\infty)$ ✓

۴)

الف) $y = \frac{۳x+۱}{x-۲} \Rightarrow R_f = \mathbb{R} - \{\frac{۳}{۱}\}$ ✓

ب) $y = \frac{۲x+۱}{x+۱} \Rightarrow R_f = \mathbb{R} - \{۲\}$ ✓

(۲)

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۵)

الف) $y = \sqrt{\frac{۲x+۴}{x+۱}} \Rightarrow R_f = \sqrt{\mathbb{R} - \{۲\}} = [۰, +\infty) - \{\sqrt{۲}\}$ ✓

ب) $y = \sqrt{\frac{۳x+۱}{۲-x}} \Rightarrow R_f = \sqrt{\mathbb{R} - \{-۳\}} = [۰, +\infty)$ ✓

(۲)

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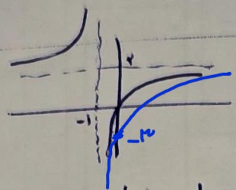
۶)

الف) $y = \frac{۲x-۳}{x+۱}$



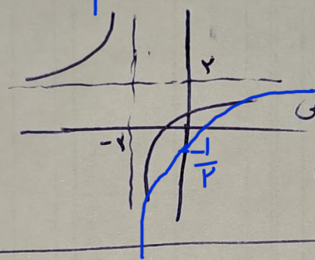
مجانب افقی $= \frac{a}{c} = ۲$

ریشه مخرج ۱- = x قائم

(۱,۵)

محل برخورد نمودار با محورهای x و y به خصوص تو امتحان نهایی

ب) $y = \frac{۲x-۱}{x+۲}$



مجانب افقی $= \frac{a}{c} = ۲$

قائم $x = -۲$

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۷)

الف) $y = \sin + \frac{۱}{\sin}$ $\quad R_f = (-\infty, -۲] \cup [۲, +\infty)$ ✓

$\sin > ۰ \Rightarrow R_f = [۲, +\infty)$

$\sin < ۰ \Rightarrow R_f = (-\infty, -۲]$

ب) $y = \frac{x^۴+۱}{x^۲} \Rightarrow x^۲ + \frac{۱}{x^۲} \Rightarrow R_f = (-\infty, -۲] \cup [۲, +\infty)$ ✓

ج) $y = \frac{\sqrt[۳]{x^۲+۱}}{\sqrt[۳]{x}} = \sqrt[۳]{x} + \frac{۱}{\sqrt[۳]{x}} \Rightarrow R_f = (-\infty, -۲] \cup [۲, +\infty)$ ✓

د) $y = \sqrt{x} + \frac{۱}{\sqrt{x}} \Rightarrow R_f = [۲, +\infty)$ ✓

(۲)

(۱۸) (۲)

الف) $y = x^2 + \frac{1}{x^2+3} = x^2 + \frac{1}{x^2+3} + 3 - 3 = x^2 + 3 + \frac{1}{x^2+3} - 3$

$x^2+3 \xrightarrow{a>0} \min \Rightarrow -\frac{b}{2a} = \frac{-0}{2(1)} = 0 \Rightarrow y_{min} = 3 \Rightarrow 3 + \frac{1}{3} - 3 = \frac{1}{3}$

$R_f = [\frac{1}{3}, +\infty)$ ✓

ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \frac{x^2+4+1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \Rightarrow x^2+4 \Rightarrow x_{min} = -\frac{b}{2a} = 0$

$y_{min} = 4$

$\Rightarrow y = \sqrt{4} + \frac{1}{\sqrt{4}} = 2 + \frac{1}{2} = \frac{5}{2} \Rightarrow R_f = [\frac{5}{2}, +\infty)$ ✓

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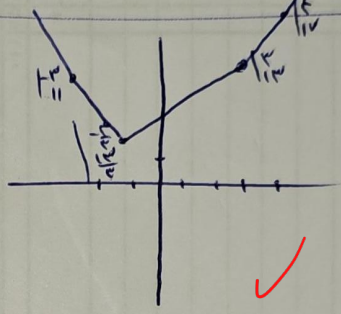
(۱۹) (۲)

$y = |x-3| + |2x+4|$

$\begin{cases} x \geq 3 : x-3+2x+4 = 3x+1 \\ -2 \leq x < 3 : -x+3+2x+4 = x+7 \\ x < -2 : -x+3-2x-4 = -3x-1 \end{cases}$



✓

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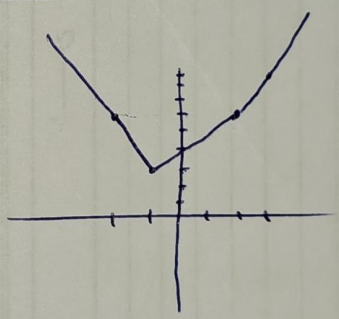
(۱۰) (۲)

الف) $y = |x-2| + |2x+2|$



$x=2 \Rightarrow y = |2-2| + |2(2)+2| = 6$

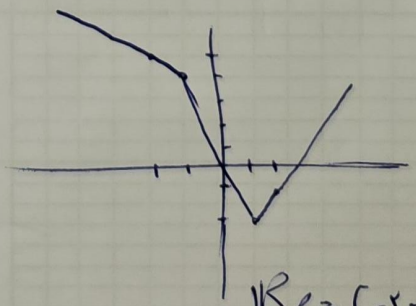
$x=-1 \Rightarrow y = |(-1)-2| + |2(-1)+2| = 3$

$x=3 \Rightarrow y = |3-2| + |2(3)+2| = 9$

$x=-3 \Rightarrow y = |(-3)-2| + |2(-3)+2| = 6$

$R_f = [3, +\infty)$ ✓

ب) $y = |2x-2| - |x+1|$



$x=1 \Rightarrow y = |2(1)-2| - |1+1| = -2$

$x=-1 \Rightarrow y = |2(-1)-2| - |-1+1| = 4$

$x=2 \Rightarrow y = |2(2)-2| - |2+1| = -1$

$x=-2 \Rightarrow y = |2(-2)-2| - |-2+1| = 5$

$R_f = [-2, +\infty)$ ✓