

الف) $y = x^3 - 3x^2 + 3x \rightarrow y = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow x = \sqrt[3]{y-1} + 1$

$\rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \rightarrow y = \frac{1}{(x-1)^2 - 1} \rightarrow y(x-1)^2 - y = 1 \Rightarrow x = \sqrt{\frac{1}{y} + 1} + 1$
 $x = -\sqrt{\frac{1}{y} + 1} + 1 \rightarrow \frac{1}{1 + \frac{1}{y} - 1} \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

الف) $x^2 - 4x + 6 = y \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{f}{a} = 2 \\ y_{\min} = y \end{cases} \rightarrow R_f = [2, +\infty)$

ب) $y = -x^2 + 4x + 3 \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = 2 \\ y_{\max} = 11 \end{cases} \rightarrow R_f = (-\infty, 11]$

ج) $y = \sqrt{x^2 - 4x + 6} \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{f}{a} = 2 \\ y_{\min} = 2 \end{cases} \rightarrow R_f = [2, +\infty)$

د) $y = \sqrt{4x - x^2} \xrightarrow{a < 0} \begin{cases} x_{\max} = 2 \\ y_{\max} = 2 \end{cases} \rightarrow R_f = [0, 2] \rightarrow R_f = [0, 2]$

الف) $y = x^3 - 3x^2 + 3x + 1 \rightarrow R_f = \mathbb{R}$

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ج) $y = \sqrt{x^3 - 3x^2 + 3x + 1} \rightarrow R_f = [0, +\infty)$

د) $y = (x^3 - 3x^2 + 3x + 1)^2 \rightarrow y = (f(x))^2 \rightarrow y \geq 0 \rightarrow R_f = [0, +\infty)$

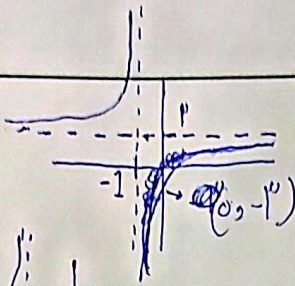
الف) $y = \frac{x^2 + 1}{x - 1} \rightarrow R_f = \mathbb{R} - \{1\}$

ب) $y = \frac{x^2 + 1}{x + 1} \rightarrow R_f = \mathbb{R} - \{-1\}$

الف) $y = \sqrt{\frac{x^2 + 1}{x + 1}} \rightarrow R_f = [0, +\infty) - \{\sqrt{2}\}$

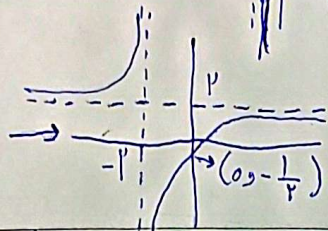
ب) $y = \sqrt{\frac{x^2 + 1}{1 - x}} \rightarrow R_f = [0, +\infty) \Rightarrow \frac{a}{c} = -1 \rightarrow \sqrt{-1} = \text{تعریف نشده}$

الف) $y = \frac{2x-1}{x+2} \rightarrow$



نقطه کنگی (0, -1/2) و جانب عمودی $x+2=0 \Rightarrow x=-2$
 $\Rightarrow \frac{a}{c} = \frac{2}{1} = 2$ جانب افقی

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نقطه کنگی (0, -1/2) و جانب عمودی $x+2=0 \Rightarrow x=-2$
 $\frac{a}{c} = \frac{2}{1} = 2$ جانب افقی

الف) $y = \sin x + \frac{1}{\sin x}$

$a < 0$
 $\frac{a}{b} > 0$

$R_f = (-\infty, -2] \cup [2, +\infty)$

منفی هم نامی

$y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$R_f = [2, +\infty)$

ب) $y = \frac{x^2+1}{x^3+1} = \frac{x^2+1}{x^3+1}$

$a < 0$
 $\frac{a}{b} > 0$

$R_f = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \frac{\sqrt[3]{x^2+1}}{\sqrt[3]{x}}$

$a < 0$
 $\frac{a}{b} > 0$

$R_f = (-\infty, -2] \cup [2, +\infty)$

الف) $y = x^2 + \frac{1}{x^2+3} = (x^2+3 + \frac{1}{x^2+3}) - 3$

$R_f = [\frac{1}{3}, +\infty)$

کمترین مقدار y
 $\frac{1}{3} + \frac{1}{3} - 3 = \frac{1}{3}$

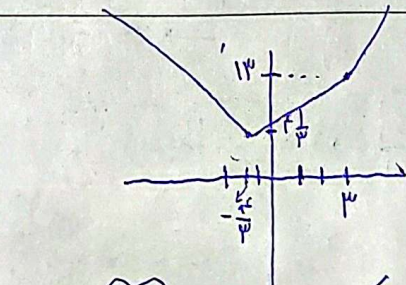
ب) $y = \frac{x^2+4}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}}$

$R_f = [\frac{4}{3}, +\infty)$

کمترین مقدار y
 $2 + \frac{1}{2} = \frac{4}{3}$

$y = |x-3| + |3x+4|$

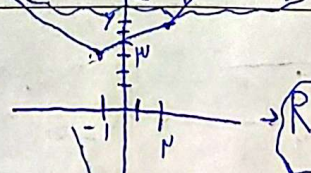
$\rightarrow \begin{cases} x-3+3x+4 & ; x > 3 \\ -x+3+3x+4 & ; -\frac{4}{3} \leq x \leq 3 \\ -x-3+3x+4 & ; x < -\frac{4}{3} \end{cases} \rightarrow \begin{cases} 4x+1 & ; x > 3 \\ 2x+7 & ; -\frac{4}{3} \leq x \leq 3 \\ 2x+1 & ; x < -\frac{4}{3} \end{cases}$



$R_f = [\frac{1}{3}, +\infty)$

الف) $y = |x-2| + |2x+2|$

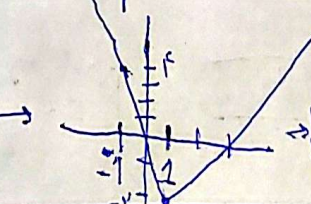
$\rightarrow \begin{cases} x-2+2x+2 & ; x > 2 \\ x-2-2x-2 & ; -1 \leq x \leq 2 \\ -x-2-2x-2 & ; x < -1 \end{cases} \rightarrow \begin{cases} 3x & ; x > 2 \\ -x-4 & ; -1 \leq x \leq 2 \\ -3x-4 & ; x < -1 \end{cases}$



$R_f = [3, +\infty)$

ب) $y = |2x-2| - |x+1|$

$\rightarrow \begin{cases} 2x-2-x-1 & ; x > 1 \\ 2-2x-x-1 & ; -1 \leq x \leq 1 \\ -2x-2-x-1 & ; x < -1 \end{cases} \rightarrow \begin{cases} x-3 & ; x > 1 \\ 1-3x & ; -1 \leq x \leq 1 \\ -3x-3 & ; x < -1 \end{cases}$



$R_f = [-2, +\infty)$