

الف) $y = x^3 - 3x^2 + 3x \rightarrow y = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow x = \sqrt[3]{y-1} + 1$

$\rightarrow R_f = \mathbb{R}$ ✓

ب) $y = \frac{1}{x^2 - 2x} \rightarrow y = \frac{1}{(x-1)^2 - 1} \rightarrow y(x-1)^2 - y = 1 \Rightarrow x = \sqrt{\frac{1}{y} + 1} + 1$

$x = -\sqrt{\frac{1}{y} + 1} + 1 \rightarrow \frac{1}{+0} = \frac{1}{-0} \rightarrow R_f = (-\infty, -1] \cup [0, +\infty)$ ✓

الف) $x^2 - 4x + 6 = y \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{c}{a} = 2 \\ y_{\min} = y \end{cases} \rightarrow R_f = [2, +\infty)$ ✓

ب) $y = -x^2 + 4x + 3 \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = 2 \\ y_{\max} = 11 \end{cases} \rightarrow R_f = (-\infty, 11]$ ✓

ج) $y = \sqrt{x^2 - 4x + 3} \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{c}{a} = 2 \\ y_{\min} = 1 \end{cases} \rightarrow R_f = [1, +\infty)$ ✓

د) $y = \sqrt{4x - x^2} \xrightarrow{a < 0} \begin{cases} x_{\max} = 2 \\ y_{\max} = 2 \end{cases} \rightarrow R_f = [0, 2]$ ✓

الف) $y = x^3 - 3x^2 + 3x + 1 \rightarrow R_f = \mathbb{R}$ ✓

ب) $y = x^3 - 3x^2 + 3x + 1 \rightarrow R_f = \mathbb{R}$ ✓

ج) $y = \sqrt{x^3 - 3x^2 + 3x + 1} \rightarrow R_f = [0, +\infty)$ ✓

د) $y = (x^3 - 3x^2 + 3x + 1)^2 \rightarrow y = (f(x))^2 \rightarrow y \geq 0 \rightarrow R_f = [0, +\infty)$ ✓

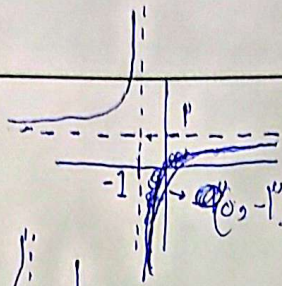
الف) $y = \frac{x^2 + 1}{x - 1} \rightarrow R_f = \mathbb{R} - \{1\}$ ✓

ب) $y = \frac{x^2 + 1}{x + 1} \rightarrow R_f = \mathbb{R} - \{-1\}$ ✓

الف) $y = \sqrt{\frac{x^2 + 1}{x + 1}} \rightarrow R_f = [0, +\infty) - \{\sqrt{2}\}$ ✓

ب) $y = \sqrt{\frac{x^2 + 1}{1 - x}} \rightarrow R_f = [0, +\infty) \Rightarrow \frac{a}{c} = -1 \rightarrow \sqrt{-1} = \text{تعریف نشده}$ ✓

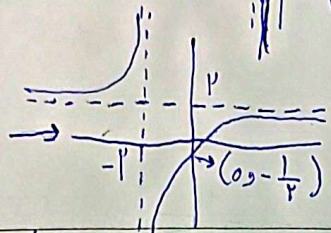
$$الف) y = \frac{2x-1}{x+2} \rightarrow$$



نقطه کنگی (0, -1/2) و جانب عمودی $x = -2$ $\Rightarrow$ $\frac{a}{c} = \frac{2}{1} = 2$ جانب افقی

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نقطه کنگی (0, -1/2) و جانب عمودی $x = -2$ $\Rightarrow$ $\frac{a}{c} = \frac{2}{1} = 2$ جانب افقی

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$$الف) y = \sin x + \frac{1}{\sin x}$$

$$\xrightarrow{a < 0} R_f = (-\infty, -2] \cup [2, +\infty)$$

منفی سینوس

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$$ب) y = \frac{x^2+1}{x^3+1} = \frac{x^2+1}{x^3+1}$$

$$\xrightarrow{a < 0} R_f = (-\infty, -2] \cup [2, +\infty)$$

$$د) y = \sqrt{x+1} + \frac{1}{\sqrt{x}}$$

$$\xrightarrow{a < 0} R_f = [2, +\infty)$$

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$$ج) y = \frac{\sqrt{x+1}}{\sqrt{x}}$$

$$\xrightarrow{a < 0} R_f = (-\infty, -2] \cup [2, +\infty)$$

$$الف) y = x^2 + \frac{1}{x^2+3} = (x^2+3 + \frac{1}{x^2+3}) - 3 \rightarrow R_f = [\frac{1}{3}, +\infty)$$

$$\xrightarrow{a < 0} R_f = [\frac{1}{3}, +\infty)$$

کمترین مقدار y $\Rightarrow \frac{1}{3} + \frac{1}{3} - 3 = \frac{1}{3}$

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$$ب) y = \frac{x^2+4}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \rightarrow R_f = [\frac{4}{3}, +\infty)$$

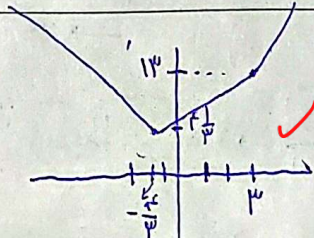
$$\xrightarrow{a < 0} R_f = [\frac{4}{3}, +\infty)$$

کمترین مقدار y $\Rightarrow 2 + \frac{1}{2} = \frac{5}{2}$

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$$y = |x-3| + |3x+4|$$

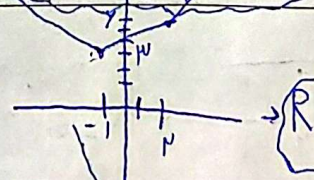
$$\rightarrow \begin{cases} x-3+3x+4 & ; x > 3 \\ -x+3+3x+4 & ; -\frac{4}{3} \leq x \leq 3 \\ -x-3-3x-4 & ; x < -\frac{4}{3} \end{cases} \rightarrow \begin{cases} 4x+1 & ; x > 3 \\ 2x+7 & ; -\frac{4}{3} \leq x \leq 3 \\ -4x-7 & ; x < -\frac{4}{3} \end{cases}$$



$$\rightarrow R_f = [\frac{4}{3}, +\infty)$$

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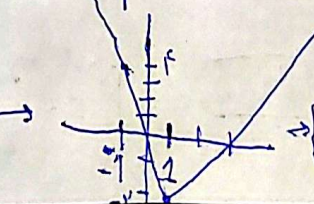
$$الف) y = |x-2| + |2x+1| \rightarrow \begin{cases} x-2+2x+1 & ; x > 2 \\ x-2-2x-1 & ; -1 \leq x \leq 2 \\ -x-2-2x-1 & ; x < -1 \end{cases} \rightarrow \begin{cases} 3x-1 & ; x > 2 \\ -x-3 & ; -1 \leq x \leq 2 \\ -3x-3 & ; x < -1 \end{cases}$$



$$\rightarrow R_f = [1, +\infty)$$

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$$ب) y = |2x-1| - |x+1| \rightarrow \begin{cases} 2x-1-x-1 & ; x > 1 \\ 1-2x-x-1 & ; -1 \leq x \leq 1 \\ -2x+1-x-1 & ; x < -1 \end{cases} \rightarrow \begin{cases} x-2 & ; x > 1 \\ -3x & ; -1 \leq x \leq 1 \\ -3x & ; x < -1 \end{cases}$$



$$\rightarrow R_f = [-2, +\infty)$$

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