[illegible]

1-

$f(x) = \frac{1}{2}^{x-2} \Rightarrow f(x+1) = \left(\frac{1}{2}\right)^{x-1} - 1$

$g = \sqrt{x^3 f(x+1)} \Rightarrow x^3 f(x+1) \geq 0 \Rightarrow x^3 \times (2^{1-x} - 1) \geq 0$

$0 \quad 1$: $- \quad + \quad -$

$D_f = [0, 1]$

2-

$f(x) = \sqrt{x + |x+2|} \Rightarrow f(-x) = \sqrt{-x + |2-x|}$

if $x \geq 2 = -x + x - 2 = -2$ زیر رادیکال باید مثبت باشد

$x \leq 2 = -2x + 2$

$-2x + 2 \geq 0 \Rightarrow 2 \geq 2x$

$D_{f(-x)} = 1 \geq x$ $\quad 1 \geq x$

3-

$\sqrt{\frac{x-1}{f(x)}} : \quad \frac{x-1}{f(x)} \geq 0$ ریشه $= x = 1$

$x = 2$

$x = 3$

$x = -4$

$-4 \quad 1 \quad 2 \quad 3$: $- \quad + \quad - \quad + \quad -$

$D(f) = (-4, 1] \cup (2, 3)$

4-

$9x^4 - 7x^2 - 4x - 3 \neq 0 \Rightarrow (9x^4 - 6x^2 + 1) - (x^2 + 4x + 4) = (3x^2 - 1)^2 - (x+2)^2$

$\Rightarrow (3x^2 + x + 1)(3x^2 - x - 3) \Rightarrow D_f = \mathbb{R} - \{$ریشه‌های $3x^2 - x - 3\}$ $\quad D_g = \mathbb{R} - \{3x^2 + ax + b$ ریشه‌ی $\}$

$\Delta < 0$ ریشه ندارد

$3x^2 - x - 3 = 3x^2 + ax + b \Rightarrow a = -1, \; b = -3$ $\quad a + b = -4 \Rightarrow (a \cdot b) = 4$

$\Rightarrow \sqrt{(a+b)} = \sqrt{4} = 2$

5-

$f(x) = x^3 + x$

$f\left(\frac{4}{x}\right) = \left(\frac{4}{x}\right)^3 + \frac{4}{x} = \frac{64}{x^3} + \frac{4}{x}$

$\Big\} \; f(x) - f\left(\frac{4}{x}\right) = x^3 + x - \frac{64}{x^3} - \frac{4}{x}$

$g = \sqrt{f(x) - f\left(\frac{4}{x}\right)} \Rightarrow f(x) - f\left(\frac{4}{x}\right) \geq 0$

$\Rightarrow \left(x^3 - \frac{64}{x^3}\right) + \left(x - \frac{4}{x}\right) \geq 0 \Rightarrow \left(x - \frac{4}{x}\right)\left(x^2 + 4 + \frac{16}{x^2}\right) + \left(x - \frac{4}{x}\right) \geq 0$

$\Rightarrow \left(x - \frac{4}{x}\right)\left(x^2 + 5 + \frac{16}{x^2}\right) \geq 0$ $\quad -2 \quad +2$: $+ \quad - \quad +$ $\quad D_g = \mathbb{R} - (-2, 2)$

$x = \frac{4}{x} \quad x^2 = 4 \quad x = \pm 2$ $\quad x^4 + 5x^2 + 16 \; \Delta < 0, \; a > 0$ همواره مثبت

[illegible] - 6

$f(x^3+x) = 2x^2-1 \Rightarrow f(1^3+1) = \frac{2-1}{1} = f(2) = 1$

$f(2)+f(10) = 8$

$\Rightarrow f(2^3+2) = 2\times 2^2 - 1 = f(10) = 7$

- 7

$f(x) = x^3 - 6x^2 + 12x$ $\quad x^3 - 3\times 2x^2 + 3\times 2^2 x - 2^3$ $\quad \cancel{x(x^2-6x+12)} = (x-2)^3 + 8$

$g(x) = x^3 + 9x^2 + 27x = (x+3)^3 - 27 = x^3 + 9\times x^2 + 3\times 3^2 \times x + 3^3 = x^3 + 9x^2 + 27x$

$f(x) = (x-2)^3 + 8 \qquad f(\sqrt[3]{2}+2) = (\sqrt[3]{2})^3 + 8 = 10$

$g(x) = (x+3)^3 - 27 \qquad g(\sqrt[3]{7}-3) = (\sqrt[3]{7}-3+3)^3 - 27 = 7 - 27 = -20$

$\frac{-20}{10} = -2$

- 8

$f(x) = \sqrt{x+4\sqrt{x-4}} = \sqrt{x-4+4\sqrt{x-4}+4} = \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2|$

$= \sqrt{x-4}+2 \qquad g(x) = \sqrt{x-4\sqrt{x-4}} = \sqrt{x-4-4\sqrt{x-4}+4} = \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2|$

$\Rightarrow |\sqrt{x-4}-2| \Rightarrow \begin{cases} \sqrt{x-4}-2 & x \geq 8 \\ -\sqrt{x-4}+2 & 4 \leq x \leq 8 \end{cases} \Rightarrow f(x)+g(x) = \begin{cases} \underset{a}{0} + \underset{b}{2}\sqrt{x-4} \\ \underset{a}{4} + \underset{b}{0}\sqrt{x-4} \end{cases}$

$x \geq 8 \Rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$

$4 \leq x < 8 \Rightarrow \frac{a+b}{c} = \frac{4+0}{-4} = -1$

- 9

$g(x) = \{(2,1)\ (1,0)\ (3,5)\ (0,4)\}$

$f(x) = \frac{x+2}{x^2-4x+3} \Rightarrow f(x) = \{(2,-4), (1, \nexists), (3, \nexists), (0, \frac{2}{3})\}$

$(x-3)(x-1)$

$\frac{g}{f} = \{(2, -\frac{1}{4}), (0, 6)\}$

- 10

$f(2x) = \{(\frac{1}{2}, 2), (\frac{3}{2}, -1), (2,2), (-\frac{1}{2}, 6)\}$

$f(x^2)\ \{(1,2), (\sqrt{3}, -1), (2,2)\}$

$2g^2(x)+1 = (-2, 19), (1, 3), (3, 9), (-1, 1)$

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2) $\frac{2f}{g}\ \{(1,-4), (3,-1), \cancel{(-1,)}\}$