

$$\left. \begin{aligned} x^y \left(\left(\frac{1}{y} \right)^{x+1-y} - 1 \right) &\geq 0 \Rightarrow x^y \left(\left(\frac{1}{y} \right)^{x-1} - 1 \right) \geq 0 \\ \left(\frac{1}{y} \right)^a \geq 1 &\Leftrightarrow a \leq 0 \end{aligned} \right\} \Rightarrow \begin{array}{c} x \mid \begin{array}{ccc} 0 & + & 1 \\ - & 0 & + \end{array} \\ x \in [0, 1] \end{array} \quad \checkmark$$

(۲)

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$$f(-x) = \sqrt{-x + |y-x|} \Rightarrow |y-x| - x \geq 0 \quad \begin{cases} x \leq y \Rightarrow y-x-x \geq 0 \Rightarrow y \geq 2x \Rightarrow x \leq \frac{y}{2} \\ x \geq y \Rightarrow x-y-x \geq 0 \Rightarrow -y \geq 0 \Rightarrow y \leq 0 \end{cases}$$

این محدوده ها را در نظر بگیریم: $1 \leq x \leq \frac{y}{2}$ و $y \leq 0$

$$\Rightarrow x \in \left[\frac{y}{2}, +\infty \right) \quad (-\infty, 1]$$

(۱, ۷۵)

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$$\frac{x-1}{f(x)} \geq 0 \quad \begin{array}{c} x \mid \begin{array}{cccc} -f & 1 & y & y \\ 0 & + & 0 & + \end{array} \\ x \in (-\infty, 1] \cup (y, y) \end{array} \xrightarrow{\text{این است}} x \in (-\infty, 1] \cup (y, y) \quad \checkmark$$

$x \in D_f \Rightarrow x \in [-\infty, y] \quad (11)$

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$$9x^6 - 17x^5 - 5x - 3 = 9x^6 + 9x^5 + 1 - x^5 - 5x - 3 = (3x^3 - 1)^2 - (x+2)^2 =$$

$$(3x^3 - 1 + x + 2)(3x^3 - 1 - x - 2) = (3x^3 + x + 1)(3x^3 - x - 3) \Rightarrow D_f = \mathbb{R} - \{P(x) \text{ های ریشه های } Q(x)\}$$

$$Q(x) = 3x^3 + x + 1 \quad P(x) = 3x^3 - x - 3$$

$$\Rightarrow \begin{cases} 3x^3 + ax + b = 3x^3 + x + 1 \\ 3x^3 - x - 3 = 3x^3 - x - 3 \end{cases} \Rightarrow \begin{cases} a=1, b=1 \\ a=-1, b=-3 \end{cases}$$

$$\Rightarrow \sqrt{-(a+b)} = \sqrt{-(-1-3)} = \sqrt{4} = 2 \quad \checkmark$$

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$$x \neq 0, f(x) - f\left(\frac{x}{2}\right) \geq 0 \Rightarrow x^y + x - \left(\frac{x}{2}\right)^y - \frac{x}{2} \geq 0$$

$$\Rightarrow x - \frac{x}{2} + x^y - \left(\frac{x}{2}\right)^y = \left(x - \frac{x}{2}\right) + \left(x - \frac{x}{2}\right) \left(x^y + x\left(\frac{x}{2}\right) + \left(\frac{x}{2}\right)^y\right) = \left(x - \frac{x}{2}\right) \left(1 + x^y + \frac{x}{2} + \frac{x^y}{2}\right) \geq 0$$

$$\Rightarrow \left(x - \frac{x}{2}\right) \left(x^y + \frac{x}{2} + \frac{x^y}{2}\right) \geq 0 \Rightarrow \left(\frac{x-y}{2}\right) \left(\frac{x^y + \frac{x}{2} + \frac{x^y}{2}}{x^y}\right) \geq 0 \Rightarrow \frac{(x-y)(x+y)}{x^y} \geq 0$$

$$\Rightarrow x \mid \begin{array}{ccc} -y & 0 & y \\ - & 0 & + \end{array} \Rightarrow x \in [-y, 0) \cup [y, +\infty) \quad \checkmark$$

(۲)

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$$x^3 + x = 10 \Rightarrow x = 2 \Rightarrow f(10) = f(2^3 + 2) = 2 \times 2^2 - 1 = \boxed{7}$$

$$x^3 + x = 7 \Rightarrow x = 1 \Rightarrow f(7) = f(1^3 + 1) = 1 \times 1^2 - 1 = \boxed{0}$$

$$f(10) + f(7) = 7 + 0 = \boxed{7} \quad \checkmark$$

$$g(x) = x^3 + 9x^2 + 14x + 14 - 14 = (x+2)^3 - 14$$

$$f(x) = x^3 - 6x^2 + 14x - 14 + 14 = (x-2)^3 + 14$$

$$\frac{g(\sqrt[3]{14} - 2)}{f(\sqrt[3]{14} + 2)} = \frac{(\sqrt[3]{14} - 2 + 2)^3 - 14}{(\sqrt[3]{14} + 2 - 2)^3 + 14} = \frac{14 - 14}{14 + 14} = \frac{0}{28} = \boxed{0} \quad \checkmark$$

$$f(x) + g(x) = \sqrt{(x-2)^2 + 14} + \sqrt{(x+2)^2 + 14} = |\sqrt{x-2} + 2| + |\sqrt{x-2} - 2|$$

$$x \geq 2 \Rightarrow f(x) + g(x) = \sqrt{x-2} + 2 + 2 - \sqrt{x-2} = 4 \Rightarrow \begin{cases} c=0 \\ c \neq 0 \end{cases} \quad \checkmark$$

$$x \geq 2 \Rightarrow f(x) + g(x) = \sqrt{x-2} + 2 + \sqrt{x-2} - 2 = 2\sqrt{x-2} \Rightarrow a=0, b=2, c=-2$$

$$\Rightarrow \frac{a+b}{c} = \frac{2}{-2} = \boxed{-1} \quad \checkmark$$

$$x^2 - 5x + 3 \neq 0 \Rightarrow (x-3)(x-1) \neq 0 \Rightarrow x \neq 1, 3$$

$$f(x) \neq 0 \Rightarrow x \neq 1, 3 \quad \text{اینو چطوری بدی؟}$$

$$\Rightarrow D_{\frac{g}{f}} = \{0, 1, 2, 3\} \cap \{x \in \mathbb{R} : x \neq 1, 3\} - \{2\} = \{0\}$$

$$f(0) = \frac{2}{3}$$

$$\Rightarrow \frac{g}{f} = \left\{ \left(0, \frac{2}{3} \right) \right\} = \boxed{\left(0, \frac{2}{3} \right)} \quad \text{و } (2, \frac{1}{3})$$

$$\text{الف) } \{(0, 0, 2), (1, 0, -1), (2, 2), (-2, 0, 4)\} \quad \checkmark \quad \begin{aligned} f(x) &= (a, b), \dots \\ f(2x) &= \left(\frac{a}{2}, b\right), \dots \end{aligned}$$

$$\text{ب) } x \geq 0 \Rightarrow f(x^2) = \{(1, 2), (-1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2)\} \quad \checkmark$$

$$\begin{aligned} f(x) &= (a, b) \\ f(x^2) &= (\sqrt{a}, b) \end{aligned} \quad \begin{aligned} x^2 &= -1 \\ x &\in \mathbb{R} \end{aligned} \quad \checkmark$$

$$\text{ج) } \{(-2, 2 \times 2 + 1), (1, 2 \times (-1)^2 + 1), (3, 2 \times 3 + 1), (-1, 2 \times 0^2 + 1)\} = \{(-2, 5), (1, 2), (3, 7), (-1, 1)\}$$

$$\text{د) } D_{\frac{f}{g}} = D_f \cap D_g - \{x \mid g(x) = 0\} = \{1, 3, -1\} - \{-1\} = \{1, 3\}$$

$$\frac{f}{g} = \left\{ \left(1, \frac{2}{3} \right), \left(3, \frac{2}{3} \right) \right\} - \left\{ \left(1, \frac{2}{3} \right), \left(3, \frac{2}{3} \right) \right\}$$

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