

$f(m) = \left(\frac{1}{r}\right)^{m-1} - 1$
 $\Rightarrow y = \sqrt{x^r f(m+1)} = ?$
 $\Rightarrow (x^r) \cdot \left(\left(\frac{1}{r}\right)^{m+1} - 1\right) = (x^r) \cdot \left(r^{-m+1} - 1\right) = (x^r) \cdot (r^{-m} \cdot r - 1) \geq 0 \xrightarrow{\text{لأن } r=2} \begin{cases} m=0 \\ m=1 \end{cases}$
 $\xrightarrow{\text{نقطة صفرية}} \begin{array}{c} 0 \quad 1 \\ - \quad + \quad - \end{array} \Rightarrow \boxed{D_f = [0, +1]} \checkmark$

$f(m) = \sqrt{x + |x + y|}$
 if $f(m) = \sqrt{m + |m + y|} \rightarrow f(-m) = \sqrt{-x + |-x + y|}$
 $\circ f(-m) = ?$
 $-x + |-x + y| \geq 0 \rightarrow |-m + y| \geq x \xrightarrow{\text{حلها}} \begin{cases} -m + y \geq x \rightarrow y \leq x + m \rightarrow m \leq 1 \\ -m + y \leq -x \rightarrow y \leq 0 \end{cases}$
 $\Rightarrow \text{الحل هو } D_f = (-\infty, +1]$

$D_f = \sqrt{\frac{x-1}{f(x)}} = ?$

رابطه را $\frac{x-1}{f(x)} > 0$ تعیین علامت

صورت $\rightarrow +$
مخرج $\begin{cases} +2 \\ +3 \\ -4 \end{cases}$

با توجه به نمودار، به ازای هر x که y آن منفی نشود.

$D_f = (-4, +1] \cup (2, 3)$

$y_1 = \frac{1}{9x^2 - 4x - 1}$
 $\Rightarrow D_{y_1} = D_{y_2} \rightarrow$ $\text{در کسر ها} \rightarrow$ مخرج ها را برابر
 $y_2 = \frac{1}{3x^2 + ax + b} \Rightarrow 9x^2 - 4x - 1 = (3x^2 + ax + b)(3x^2 - x - 1)$
 $\sqrt{-(a+b)} = ? \quad 3x^2 + ax + b = 3x^2 - x - 1 \quad \oplus \text{ only } \leftarrow$
 $\Rightarrow \begin{cases} a = -1 \\ b = -1 \end{cases} \Rightarrow \sqrt{-(a+b)} = \sqrt{-(-1-1)} = \sqrt{2}$

$f(x) = x^r + x \rightarrow f(x) = x(x^{r+1})$
 $y = \sqrt{f(x) - f(\frac{\epsilon}{n})} \xrightarrow{\text{substituting}} f(x) > f(\frac{\epsilon}{n}) \Rightarrow x(x^{r+1}) > \frac{\epsilon}{n} \left(\frac{14}{x^r} + 1 \right)$
 $\Rightarrow x(x^{r+1}) > \frac{4\epsilon + \epsilon x^r}{x^r} \Rightarrow x^r + x - \frac{4\epsilon + \epsilon x^r}{x^r} > 0 \Rightarrow x^r + x^r - \frac{\epsilon x^r}{x^r} - 4\epsilon > 0$
 $\Rightarrow \begin{cases} x = r \\ x = -r \end{cases} \Rightarrow \frac{-r}{+} \frac{+r}{-} + \Rightarrow \phi = (-\infty, -r] \cup [r, +\infty)$

$$\begin{array}{cccc} - & + & - & + \\ \hline - & + & - & + \end{array}$$

$$[-r, 0) \cup [r, +\infty)$$

$$f(x^r + x) = r x^{r-1}$$

$$f(r) + f(10) = ? \rightarrow \begin{cases} x^r + x = r \rightarrow x = 1 \xrightarrow{r x^{r-1}} r(1)^{r-1} = 1 \\ x^r + x = 1 \rightarrow x = r \xrightarrow{r x^{r-1}} r(r)^{r-1} = r^r = 1 \end{cases}$$

$$\Rightarrow \begin{cases} f(r) = 1 \\ f(10) = r \end{cases} \rightarrow f(r) + f(10) = 1 + r = \boxed{\wedge} \checkmark$$

$$f(x) = x^r - 4x^r + 12x \rightarrow f(x) = x^r - 4x^r + 12x = (x-r)^r + 1$$

$$g(x) = x^r + 9x^r + 2rx \rightarrow g(x) = x^r + 9x^r + 2rx = (x+r)^r - 2r$$

$$\frac{g(\sqrt[r]{r} - r)}{f(\sqrt[r]{r} + r)} \Rightarrow \begin{cases} g(\sqrt[r]{r} - r) = (\sqrt[r]{r} - r + r)^r - 2r = r - 2r = -r \\ f(\sqrt[r]{r} + r) = (\sqrt[r]{r} + r - r)^r + 1 = r + 1 = 10 \end{cases}$$

$$\hookrightarrow \frac{g(\sqrt[r]{r} - r)}{f(\sqrt[r]{r} + r)} = \frac{-r}{10} = \boxed{-\frac{r}{10}} \checkmark$$

$$f(x) = \sqrt{x + \sqrt{x-r}}$$

$$g(x) = \sqrt{x - \sqrt{x-r}}$$

$$f(x) + g(x) = a + b\sqrt{x+c}$$

$$\frac{a+b}{c} = ?$$

$$\begin{aligned} &\xrightarrow{m=r} \begin{cases} x=r \\ x=\omega \end{cases} \rightarrow \begin{cases} f(r) + g(r) = \sqrt{r + \sqrt{r-r}} + \sqrt{r - \sqrt{r-r}} = a + b\sqrt{r+c} = r \\ f(\omega) + g(\omega) = \sqrt{\omega + \sqrt{\omega-r}} + \sqrt{\omega - \sqrt{\omega-r}} = a + b\sqrt{\omega+c} = r \end{cases} \\ &\Rightarrow a + b\sqrt{\omega+c} = a + b\sqrt{r+c} = r \Rightarrow \sqrt{\omega+c} = \sqrt{r+c} \xrightarrow{\text{مربع}} |\omega+c| = |r+c| \\ &\Rightarrow \begin{cases} r+c = -\omega-c \quad (\times) \\ -r-c = \omega+c \quad (\checkmark) \end{cases} \rightarrow -r-c = \omega+c \rightarrow 2c = 1 \rightarrow \boxed{c = \frac{1}{2}} \end{aligned}$$

$$f(x) = \frac{x+r}{x^r - rx + r} \rightarrow f(x) = \frac{x+r}{(x-1)(x-r)} \Rightarrow D_f = \mathbb{R} - \{1, r\}$$

$$g(x) = \left\{ \left(\frac{r}{r}, 1 \right), \left(\frac{1}{1}, 0 \right), \left(\frac{r}{r}, \omega \right), \left(\frac{0}{0}, r \right) \right\}$$

$$g/f = ? \rightarrow D_f \cap D_g = \{r, 0\}$$

$$\hookrightarrow g/f = \left\{ \left(r, -\frac{1}{r} \right), \left(0, \frac{r}{r} \right) \right\} \Rightarrow \boxed{g/f = \left\{ \left(r, -\frac{1}{r} \right), \left(0, 1 \right) \right\}} \checkmark$$

$$f(x) = \left\{ (1, r), (r, -1), (r, r), (-\frac{1}{r}, r) \right\}$$

$$g(x) = \left\{ (-r, r), (1, -1), (r, r), (-1, 0) \right\}$$

$$a) f(rx) \rightarrow \text{دالة مركبة} \rightarrow f(rx) = \left\{ \left(\frac{1}{r}, r \right), \left(\frac{r}{r}, -1 \right), \left(r, r \right), \left(-\frac{1}{r}, r \right) \right\} \checkmark$$

$$b) f(x^r) \rightarrow \text{دالة مركبة} \rightarrow f(x^r) = \left\{ (1, r), (\sqrt[r]{r}, -1), (r, r), (\sqrt[r]{-1}, r), (-1, r), (-\sqrt[r]{r}, -1), (-r, r) \right\} \checkmark$$

$$c) r g^r(x) + 1 \rightarrow r g^r(x) + 1 = \left\{ (-r, r(r^r+1)), (1, r(-1)^r+1), (r, r(r^r+1)), (-1, r(0)^r+1) \right\} \Rightarrow \left\{ (-r, 19), (1, 3), (r, 9), (-1, 1) \right\} \checkmark$$

$$d) r f/g \rightarrow D_{f/g} = \{ \pm 1, r \}$$

$$\Rightarrow r f/g = \left\{ \left(-1, \frac{r(r)}{-1} \right), \left(+1, \frac{r(r)}{-1} \right), \left(r, \frac{r(-1)}{r} \right) \right\} = \left\{ (-1, -r), (r, -1) \right\} = r f/g \checkmark$$