

$$f(x) = \left(\frac{1}{x}\right)^{n-1} - 1 \quad y = \sqrt{x^n f(x+1)} \rightarrow D_f = ?$$

$$\Rightarrow x^n x f(x+1) \geq 0 \rightarrow x^n \left(\left(\frac{1}{x}\right)^{n-1} - 1\right) \geq 0 \Rightarrow \frac{1}{-x + x^n} \geq 0$$

$$\Rightarrow D_f = [0, 1]$$

$$f(x) = \sqrt{x + |x+1|} \quad f(-x) \rightarrow D_f = ?$$

$$f(-x) = \sqrt{-x + |-x+1|} \rightarrow -x + |-x+1| \geq 0 \begin{cases} -x + x + 1 & x \geq 1 \\ -x - x + 1 & x < 1 \end{cases}$$

$$\Rightarrow \begin{cases} 1 & x \geq 1 \\ -x + 1 & x < 1 \end{cases} \rightarrow -x + 1 \geq 0 \rightarrow -x \geq -1 \rightarrow x \leq 1 \Rightarrow D_f = (-\infty, 1]$$

$$y = \sqrt{\frac{x-1}{f(x)}} \rightarrow \frac{x-1}{f(x)} \geq 0 \quad \text{for } \begin{cases} x=1 \rightarrow y=0 \\ x=2 \rightarrow y=0 \\ x=3 \rightarrow y=0 \end{cases}$$

$$\Rightarrow \frac{1}{-x+1} \geq 0 \rightarrow D_f = (-1, 1] \cup (3, 4)$$

$$y_1 = \frac{1}{9x^3 - 4x^2 - 4x - 3} \quad y_2 = \frac{1}{10x^3 + 4x + 2}$$

$$\Rightarrow 9x^3 - 4x^2 - 4x - 3 = (3x^2 + mx + n)(3x + k) = 9x^3 + 3kx^2 + 3mx + mnx + 3n$$

در اینجا $3k = -4$ و $3n = -3$ را داریم.

$$f(x) = x^3 + x \quad y = \sqrt{f(x) - f\left(\frac{x}{2}\right)} \rightarrow x^3 + x - \left(\frac{x^3}{8} + \frac{x}{2}\right) = \frac{7x^3}{8} + \frac{x}{2} = \frac{x^3}{8} \geq 0$$

$$\rightarrow \frac{(x^3 - 1)(x^3 + 4x^2 + 4x + 1)}{x^3} \geq 0 \rightarrow \frac{1}{-x^3 + 4x^2 - 4x + 1} \geq 0 \rightarrow D_f = [-1, 0) \cup [1, +\infty)$$

$$f(x^3+x) = |x^3-1| \quad f(1)+f(10)=?$$

$$x^3+x=10 \rightarrow x^3+x-10=0 \xrightarrow{x(1)+1-10=0} x=1 \rightarrow f(10)=|1^3-1|=0$$

$$x^3+x=1 \rightarrow x^3+x-1=0 \xrightarrow{x(1)+1-1=0} x=1 \rightarrow f(1)=|1^3-1|=0$$

$$\left. \begin{array}{l} f(10)=0 \\ f(1)=0 \end{array} \right\} f(1)+f(10)=0+0=0$$

$$f(x)=x^3-7x^2+12x \quad g(x)=x^3+9x^2+12x \quad \frac{g(\sqrt[3]{1}-1)}{f(\sqrt[3]{1}+1)}=?$$

$$g(x)=(x+3)^3-12 \rightarrow g(\sqrt[3]{1}-1)=(\sqrt[3]{1}-1+3)^3-12=(\sqrt[3]{1}+2)^3-12=10$$

$$f(x)=(x-2)^3+12 \rightarrow f(\sqrt[3]{1}+1)=(\sqrt[3]{1}+1-2)^3+12=(-1)^3+12=11$$

$$\rightarrow \frac{g(\sqrt[3]{1}-1)}{f(\sqrt[3]{1}+1)} = \frac{10}{11}$$

$$f(x)=\sqrt{x-1} \quad g(x)=\sqrt{x-1} \quad x \geq 1$$

$$t=\sqrt{x-1} \rightarrow x=t^2+1 \Rightarrow f(x)=\sqrt{t^2+1-1}=t \quad g(x)=\sqrt{t^2+1-1}=t$$

$$g(x)=\sqrt{t^2+1-1}=t \quad f(x)+g(x)=t+t=2t=2\sqrt{x-1}$$

$$t \geq 1 \rightarrow 1+t+t-1=2t=2\sqrt{x-1} \Rightarrow f(x)+g(x)=2\sqrt{x-1} \Rightarrow a=0, b=2, c=-1$$

$$0 < t < 1 \rightarrow t+1+t-1=2t=2\sqrt{x-1}$$

$$f(x)=\frac{x+1}{x^2-1}, \quad g(x)=\left\{ \left(\frac{1}{2}, 1 \right), \left(1, 0 \right), \left(\frac{3}{2}, 1 \right), \left(2, 1 \right) \right\}$$

$$\frac{g}{f} \Rightarrow \left\{ \left(0, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right), \left(1, \frac{1}{2} \right), \left(\frac{3}{2}, \frac{1}{2} \right) \right\}$$

$$D_f \rightarrow f(x)=\frac{x+1}{(x-1)(x-3)} \Rightarrow D_f = \mathbb{R} - \{1, 3\} \Rightarrow f(0)=\frac{1}{3}, f(1)=\frac{1}{2}$$

$$\rightarrow \frac{g}{f} = \left\{ \left(0, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right) \right\}$$

$$f(x)=\{(1,2), (3,-1), (5,2), (-1,2)\} \quad g(x)=\{(-2,3), (1,-1), (3,2), (-1,0)\}$$

$$الف) f(2x)=\left\{ \left(\frac{1}{2}, 2 \right), \left(\frac{3}{2}, -1 \right), \left(\frac{5}{2}, 2 \right), \left(-\frac{1}{2}, 2 \right) \right\}$$

$$ب) f(x^2)=\{(1,2), (4,2), (9,-1), (16,-1), (25,2), (36,2)\}$$

$$ج) f(g(x)+1)=\{(-2,3), (1,3), (3,3), (-1,3)\}$$

$$د) \frac{f}{g}=\{(1,-4), (3,-1)\}$$

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