نام و نام خانوادگی: مُوانِر شیرانی

پاسخنامه تشریحی تکلیف شماره ۴ کلاس یازدهم پسر A

۱

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \longrightarrow \frac{x-1}{x}-\frac{x}{x-1}\geq 0 \longrightarrow \frac{x^2-2x+1-x^2}{x^2-x}\geq 0 \to \frac{-2x+1}{x^2-x}\geq 0$

$\to \frac{-2x+1}{x(x-1)}\geq 0 \rightsquigarrow$ $\begin{array}{ccccccc} & 0 & & \frac{1}{2} & & 1 & \\ + & \text{ت ن} & - & 0 & + & \text{ت ن} & - \end{array}$ $\rightsquigarrow D_f=(-\infty\ \ 0)\cup[\frac{1}{2}\ \ 1)$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \longrightarrow \begin{cases} x\neq 0,\ x\neq -1,\ x\neq 1,\ x\neq -2 \rightsquigarrow D_f=R-\{-2,-1,0,1,\frac{5}{4}\} \\ \frac{3}{x-1}+\frac{1}{x+2}\neq 0 \longrightarrow \frac{3x+6+x-1}{(x^2-1)(x+2)}\neq 0 \rightsquigarrow \frac{4x-5}{(x-1)(x+2)} \to x\neq\frac{5}{4} \end{cases}$

۲

الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)} \rightsquigarrow \underbrace{((\frac{1}{3})^x-9)}_{\text{ریشه}\to -2}\ \underbrace{(32-4^x)}_{\text{ریشه}\to\frac{5}{2}}\geq 0 \rightsquigarrow$ $\begin{array}{ccccc} & -2 & & \frac{5}{2} & \\ + & 0 & - & 0 & + \end{array}$

$\rightsquigarrow D_f=R-[-2\ \ \frac{5}{2}]$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \rightsquigarrow -\sqrt[4]{x-1}+3=\sqrt{y+1} \xrightarrow{1} -\sqrt[4]{x-1}+3\geq 0 \rightsquigarrow \sqrt[4]{x-1}\leq 3$

$\rightsquigarrow x-1\leq 81 \rightsquigarrow x\leq 82$

$\xrightarrow{2} x-1\geq 0 \rightsquigarrow x\geq 1$

$\longrightarrow D_f=[1\ ,\ 82]$

۳

$f(x)=\dfrac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \rightsquigarrow$

(1) $x^2-x-2 > 0 \longrightarrow (x+1)(x-2)>0 \to$ $\begin{array}{ccccc} & -1 & & 2 & \\ + & 0 & - & 0 & + \end{array}$

(2) $x^2-1\geq 0 \longrightarrow (x-1)(x+1)\geq 0 \rightsquigarrow$ $\begin{array}{ccccc} & -1 & & 1 & \\ + & 0 & - & 0 & + \end{array}$

(3) $\sqrt{x^2-1}+1\neq 0 \longrightarrow$ همیشه برقرار

$\longrightarrow D_f=(-\infty\ \ -1)\cup(2\ \ +\infty)$

۴

$f=\sqrt{-x^2+ax+3}$ $\qquad D_f=[-2\ \ b]$

$\xrightarrow{\text{ریشه عبارت } -2} -4-2a+3=0 \rightsquigarrow -2a-1=0 \rightsquigarrow a=-\frac{1}{2} \rightsquigarrow f(x)=\sqrt{-x^2-\frac{1}{2}x+3}$

$-x^2-\frac{1}{2}x+3\geq 0 \rightsquigarrow -(x+2)(x+1.5)\geq 0$ $\begin{array}{ccccc} & -2 & & 1.5 & \\ - & | & + & | & - \end{array}$ $\rightsquigarrow D_f=[-2, 1.5]$

$\rightsquigarrow b=1.5\ ,\ a=-\frac{1}{2} \longrightarrow a+b=1$

۵

$f(x)=\begin{cases} 3x-2 & ; x\geq 1 \\ 2x+3 & ; x<1 \end{cases}$ $\qquad g(x)=\sqrt{f(x)-x}$

$(1)\cup(2)=[-3\ \ +\infty)$ $\qquad (2)\ \{x\geq 1\}$

$\rightsquigarrow g(x)=\begin{cases} \sqrt{3x-2-x} & x\geq 1 \\ \sqrt{2x+3-x} & x<1 \end{cases} \rightsquigarrow g(x)=\begin{cases} \sqrt{2x-2} & x\geq 1 \rightsquigarrow x\geq 1 \\ \sqrt{x+3} & x<1 \rightsquigarrow x\geq -3 \end{cases}$

$(1)\ \{-3\leq x<1\}$

6

$$f(x)=\begin{cases}(a+1)(x+2) & x>1\\ 3a+2x & x\le 1\end{cases} \qquad 2f(5)=f(-2)+a$$

$$\xrightarrow{f(-2)=3a-4} 2f(5)=3a-4+a \rightsquigarrow 2(a+1)(7)=4a-4 \rightsquigarrow 14a+14=4a-4$$

$$\rightsquigarrow 10a=-18 \rightsquigarrow \underline{a=-1.8}$$

$$f(2-\sqrt{3})+f(2+\sqrt{3}) \qquad f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$$

$$f(2+\sqrt{3})=\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}\times\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}}+2 \rightsquigarrow \sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}+2$$

$$\sqrt{\left(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}\right)^2} \longrightarrow \sqrt{2+\sqrt{3}+2-\sqrt{3}+2} \Rightarrow \sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}=\sqrt{6} \Big\} \to f(2+\sqrt{3})=\sqrt{6}+2$$

$$2-\sqrt{3}=2-\sqrt{3}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{1}{2+\sqrt{3}} \longrightarrow f(2-\sqrt{3})=f\left(\frac{1}{2+\sqrt{3}}\right) \rightsquigarrow f(2-\sqrt{3})=f(2+\sqrt{3})$$

$$\rightsquigarrow f(2-\sqrt{3})+f(2+\sqrt{3})=2f(2+\sqrt{3})=2(\sqrt{6}+2)=2\sqrt{6}+4$$

8

$$2f(x)-3f(-x)=4x^2-x \xrightarrow{\times 2} 4f(x)-6f(-x)=8x^2-2x$$

$$\xrightarrow{x=-x} 2f(-x)-3f(x)=4x^2+x \xrightarrow{\times 3} 6f(-x)-9f(x)=12x^2+3x$$

$$-5f(x)=20x^2+x$$

$$\rightsquigarrow -5f(x)=20x^2+x \rightsquigarrow f(x)=-4x^2-\frac{1}{5}x$$

9

$$(x+2)f(x)-3x\,f(x+2)=4x^2-mx+3m-1$$

$$\xrightarrow{x=0} 2f(0)-3(0)(f(2))=3m-1 \longrightarrow 2f(0)=3m-1 \xrightarrow{\times 3} 6f(0)=9m-3$$

$$\xrightarrow{x=-2} 0f(-2)+6f(0)=16+2m+3m-1 \longrightarrow 6f(0)=5m+15$$

$$\ominus \quad 0=4m-18 \rightarrow \underline{m=4.5}$$

$$\longrightarrow 2f(0)=13.5-1 \rightsquigarrow f(0)=6.25$$

$$f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x}$$

$$\xrightarrow{x=-1} f(-1)+f(-1)=\frac{3+12+3}{-1} \to 2f(-1)=-18 \longrightarrow \underline{f(-1)=-9}$$ 10