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الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$

~~$x \neq 0$~~
 ~~$x \neq 1$~~ ~~$x \neq -1$~~

$\frac{(x-1)(x-1)}{(x-1)(x)} - \frac{x}{(x-1)x} \geq 0$

~~$\frac{x^2-1}{x^2-x} - \frac{x}{x^2-x} \geq 0$~~
 ~~$\frac{x^2-1-x}{x^2-x} \geq 0$~~

$\frac{x^2+1-2x-x^2}{x^2-x} \geq 0 \Rightarrow \frac{1-2x}{x(x-1)} \geq 0$

~~$\frac{1-2x}{x(x-1)} \geq 0$~~

$\Rightarrow (-\infty, 0) \cup [1, 1)$

$\frac{0}{-} \frac{1}{+} \frac{1}{-}$

$\Rightarrow (-\infty, 0) \cup [1, 1)$

ب) $f(x) = \frac{1}{x+1} - \frac{2}{x}$

$= \frac{x-2x-2}{x(x+1)} = \frac{-x-2}{x(x+1)}$

$\frac{-x-2}{x(x+1)} \neq 0$ ①

$(x-1)(x+2) \neq 0$ ②

$\frac{x+2}{(x-1)(x+2)} \neq 0$ ③

① $\Rightarrow x \neq \{-1, 0\}$

② $\Rightarrow x \neq \{1, -2\}$

③ $\Rightarrow x \neq \{-\frac{2}{3}\}$

① ② ③ $\Rightarrow x = \{-1, 0, 1, -2, -\frac{2}{3}\}$

الف) $f(x) = \sqrt{(\frac{1}{x}-9)(x^2-4)}$

$\Rightarrow x = -2$ $x = 2$

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منفی شود

$\Rightarrow x \geq 0$

$\frac{-2}{+} \frac{2}{-}$ ①

$(-\infty, -2] \cup [2, +\infty)$

① ② $\Rightarrow (2, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1}$

$\Rightarrow \sqrt{x-1} \geq 0 \Rightarrow x \geq 1$

If $3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$

$f(x) = \log_e \frac{x^2-x-2}{\sqrt{x^2-1}+1}$

$\frac{x^2-x-2}{(x-2)(x+1)}$

$\frac{-1}{+} \frac{2}{-}$

$(-\infty, -1) \cup (2, +\infty)$ ①

$\sqrt{x^2-1}+1 \neq 0$

$x^2-1 \geq 0 \Rightarrow (x-1)(x+1) \geq 0$

$\frac{-1}{+} \frac{1}{-}$

$(-\infty, -1] \cup [1, +\infty)$ ②

① ② $= (-\infty, -1] \cup [1, +\infty)$

$$f(a) = \sqrt{a^2 + a + 1} \quad \text{جواب} \quad \begin{array}{c} -2 \quad b \\ - \quad + \quad - \end{array} \Rightarrow -4 - 2a + 1 = 0 \Rightarrow a = -\frac{1}{2}$$

$$a+b = -\frac{1}{2} + \frac{1}{2} = 0 \quad \text{برای } b \text{ با } a \text{ مضرب} \Rightarrow -b^2 - \frac{1}{2}b + 1 = 0$$

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$$b = \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4}}{-2} = \frac{\frac{1}{2} \pm \frac{\sqrt{3}}{2}}{-2} \Rightarrow b = \frac{-1 \pm \sqrt{3}}{4}$$

$$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0$$

$$f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 2 & x < 1 \end{cases} \Rightarrow f(x) - x = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 2 & x < 1 \end{cases}$$

$$f(x) = \begin{cases} (x+1)(x+2) & x \geq 1 \\ x^2 + 2 & x < 1 \end{cases}$$

$$2(x+1) \cdot x = -4 + 2x + 2$$

$$4x + 2 = -4 + 2x + 2 \Rightarrow 2x = -8 \Rightarrow x = -4$$

$$f(x) = \sqrt{x+1} + 2 = \sqrt{x+1} + 2 \Rightarrow f(x) = \sqrt{x+1} + 2$$

$$(2+\sqrt{3})(2-\sqrt{3}) = 1 \Rightarrow 2+\sqrt{3} = \frac{1}{2-\sqrt{3}} \Rightarrow f(2+\sqrt{3}) = f\left(\frac{1}{2-\sqrt{3}}\right) = 2f(2-\sqrt{3})$$

$$\begin{aligned} 2 \times 2f(x) - 2f(-x) &= 4x^2 - 4 = 4f(x) - 4f(-x) = 4x^2 - 4 \\ 2 \times 2f(-x) - 2f(x) &= 4x^2 + 4 = 4f(-x) - 4f(x) = 4x^2 + 4 \\ -2f(x) &= 4x^2 + 4 \Rightarrow f(x) = -2x^2 - 2 \end{aligned}$$

$$2f(0) = \dots \Rightarrow 4f(0) = 9m - 1$$

برای $x=0$

$$4f(0) = 16 + 4m$$

برای $x=0$

$$4f(0) = 16 + 4m \Rightarrow f(0) = 4 + m = 9m - 1 \Rightarrow m = \frac{5}{8}$$

نہایتی $n = -1$

$$f(-1) + f(-1) = \frac{2 + 12 + 10}{-1}$$

$$\Rightarrow 2f(-1) = -14 \Rightarrow f(-1) = -7$$

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