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11.

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$

$\frac{(x-1)(x-1)}{(x-1)(x)} - \frac{x}{(x-1)x} \geq 0$

$\frac{x^2 + 1 - 2x - x^2}{x^2 - x} \geq 0 \Rightarrow \frac{1 - 2x}{x(x-1)} \geq 0$

~~$(-\infty, 0) \cup [1, +\infty)$~~

$(-\infty, 0) \cup [1, +\infty)$ ✓

$(-\infty, 0) \cup [1, +\infty)$

ب) $f(x) = \frac{1}{x+1} - \frac{2}{x}$

$\frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{2}{x-1} + \frac{1}{x+2}} = \frac{\frac{x - 2x - 2}{x(x+1)}}{\frac{2x + x + 2}{(x-1)(x+2)}} = \frac{-x-2}{x(x+1)} \cdot \frac{(x-1)(x+2)}{3x+3}$

$\frac{-x-2}{x(x+1)} \cdot \frac{(x-1)(x+2)}{3x+3} \neq 0$ ①

$\frac{(x-1)(x+2)}{(x-1)(x+2)} \neq 0$ ②

$\frac{3x+3}{(x-1)(x+2)} \neq 0$ ③

① $\Rightarrow x \neq \{-1, 0\}$

② $\Rightarrow x \neq \{1, -2\}$

③ $\Rightarrow x \neq \{-\frac{3}{2}\}$

① ② ③ $\Rightarrow \mathbb{R} - \{-1, 0, 1, -2, -\frac{3}{2}\}$ ✓

الف) $f(x) = \sqrt{(\frac{1}{x} - 9)(x^2 - 4)}$

$\frac{1}{x} - 9 \geq 0 \Rightarrow x \leq -\frac{1}{9}$

$x^2 - 4 \geq 0 \Rightarrow x \leq -2$ or $x \geq 2$

$\frac{-1}{x} - 9 \geq 0 \Rightarrow \frac{-1 - 9x}{x} \geq 0$ ①

$(-\infty, -\frac{1}{9}] \cup [2, +\infty)$ ✓

① ② $\Rightarrow [2, +\infty)$

$-\frac{1}{9}$

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 $\Rightarrow x \neq 0$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1}$

$\sqrt{x-1} \geq 0 \Rightarrow x \geq 1$ ✓

If $3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$ ✓

$\rightarrow D = [1, 10]$

$f(x) = \log_e \frac{x^2 - x - 2}{\sqrt{x^2 - 1} + 1}$

$\frac{x^2 - x - 2}{(x-2)(x+1)} \geq 0$

$\frac{-1}{x} - 9 \geq 0 \Rightarrow \frac{-1 - 9x}{x} \geq 0$

$(-\infty, -\frac{1}{9}] \cup [2, +\infty)$ ①

$\sqrt{x^2 - 1} + 1 \neq 0$ always

$x^2 - 1 \geq 0 \Rightarrow (x-1)(x+1) \geq 0$

$\frac{-1}{x} - 9 \geq 0 \Rightarrow \frac{-1 - 9x}{x} \geq 0$

$(-\infty, -\frac{1}{9}] \cup [2, +\infty)$ ②

① ② $\Rightarrow (-\infty, -\frac{1}{9}] \cup [2, +\infty)$ ✓

$f(a) = \sqrt{a^2 + a + 1}$ بازای $a = -1$
 $df = [-1, b]$ بازای $a = -1$ $\Rightarrow -1 - 2a + 1 = 0 \Rightarrow -2a = 0 \Rightarrow a = 0$ (2)

$a + b = -\frac{1}{2} + b = 1 \Rightarrow b = \frac{3}{2}$
 $b = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{4}} = \frac{1}{2} \pm \frac{0}{2} \Rightarrow b = \frac{1}{2}$
 $b > -\frac{1}{2}$

ساده‌ترین حالت

$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0$
 $f(x) = \begin{cases} x^2 - 2x & x \geq 1 \\ x^2 + 2x & x < 1 \end{cases} \Rightarrow f(x) - x = \begin{cases} x^2 - 3x & x \geq 1 \\ x^2 + x & x < 1 \end{cases}$
 $x^2 - 3x \geq 0 \Rightarrow x \geq 3$
 $x^2 + x \geq 0 \Rightarrow x \geq 0$
 $UV = [-3, +\infty)$

$f(x) = \begin{cases} (x+1)(x+2) & x \geq 1 \\ x^2 + 2x & x \leq 1 \end{cases}$
 $2(x+1) \cdot x = -1 + 2x + x^2$
 $1(x+1) = x^2 - 1 \Rightarrow x = -1$

$f(x) = \sqrt{x + \frac{1}{x}} + 2 \Rightarrow f(x) = \sqrt{x + \frac{1}{x} + 4} + 2$
 $(2 + \sqrt{x})(2 - \sqrt{x}) = 1 \Rightarrow 4 - x = 1 \Rightarrow x = 3$
 $f(2 + \sqrt{x}) = \sqrt{2 + \sqrt{x} + \frac{1}{2 + \sqrt{x}}} + 2$
 $(2 + \sqrt{x})(2 - \sqrt{x}) = 1 \Rightarrow 2f(2 + \sqrt{x}) = 2 + \sqrt{x}$

$2 \times 2f(x) - 2f(-x) = 4x^2 - x = 4f(x) - 2f(-x) = 1x^2 - 2x$
 $2 \times 2f(-x) - 2f(x) = 4x^2 + x = 4f(-x) - 2f(x) = 1x^2 + 2x$
 $-2f(x) = 2x^2 + x \Rightarrow f(x) = -x^2 - \frac{x}{2}$

$2f(0) = \dots \Rightarrow 4f(0) = 9m - 1$
 $4f(0) = 16 + 2m$
 $9m - 1 = 16 + 2m \Rightarrow 7m = 17 \Rightarrow m = \frac{17}{7}$

نہایتی $n = -1$

$$f(-1) + f(-1) = \frac{1 + 12 + 12}{-1} \Rightarrow 2f(-1) = -11 \Rightarrow f(-1) = -\frac{11}{2} \quad (1)$$

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