

الف) $\frac{x-1}{x} - \frac{x}{x-1} = \frac{-x^2+1}{x^2-x}$

$\frac{-x^2+1}{x^2-x} \geq 0$ $x \mid \begin{array}{ccc} 0 & \frac{1}{x} & 1 \\ + & - & + \end{array}$

$x \in (-\infty, 0) \cup [\frac{1}{x}, 1)$

ب) $x+1 \neq 0 \rightarrow x \neq -1$, $x \neq 0$ ①

$x-1 \neq 0 \rightarrow x \neq 1$, $x+2 \neq 0 \rightarrow x \neq -2$ ②

$x+5 \neq 0 \rightarrow x \neq -5$ ③

$\frac{1}{x+1} - \frac{2}{x} = \frac{-(x-1)(x+2)(-x-2)}{(x+5)(x)(x+1)}$

① ∪ ② ∪ ③ $\rightarrow x \in \mathbb{R} - \{-1, 0, 1, -2, -5\}$

1

الف) $(\frac{1}{x})^x - 9 = 0 \rightarrow x = -2$

$x^2 - 4^x = 0 \rightarrow x = \frac{5}{2}$

$((\frac{1}{x})^x - 9)(x^2 - 4^x) \geq 0$

$x \mid \begin{array}{ccc} -2 & \frac{5}{2} & \\ + & - & + \end{array}$ $x \in (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب)

$x-1 \geq 0 \rightarrow x \geq 1 \rightarrow x \in [1, +\infty)$

$y = \sqrt{x-1} - 4\sqrt{x-1} + 8$

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$\log_2^{(x^2-x-2)} \rightarrow x^2-x-2 > 0 \rightarrow (x-2)(x+1) > 0$ $x \mid \begin{array}{ccc} -1 & 2 & \\ + & - & + \end{array}$

$x \in (-\infty, -1) \cup (2, +\infty)$ ① $\sqrt{x^2-1} + 1 \neq 0$ ✓

$x^2-1 \geq 0$ $x \mid \begin{array}{ccc} -1 & 1 & \\ + & - & + \end{array}$ $x \in (-\infty, -1] \cup [1, +\infty)$ ②

① ∩ ② $\rightarrow x \in (-\infty, -1) \cup (2, +\infty)$

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$\begin{array}{c} a \\ b \end{array} \rightarrow x = -2 \rightarrow 3-2a-4=0 \rightarrow a = -\frac{1}{2} \rightarrow \sqrt{3-\frac{x}{2}-x^2} = f(x)$

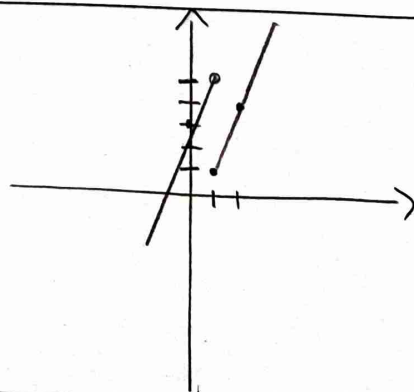
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$\rightarrow a+b = x_1+x_2 = -\frac{b}{2a} = -\frac{-\frac{1}{2}}{-2} = -\frac{1}{4}$

4

$f(x) - x \geq 0 \rightarrow f(x) \geq x$

$\left. \begin{array}{l} x=1 \rightarrow y=1 \\ x=2 \rightarrow y=4 \\ x=0 \rightarrow y=3 \end{array} \right\} x \in [-3, 1) \cup [2, +\infty)$



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$$\left. \begin{aligned} 2f(\omega) &= f(-1) + a \\ f(\omega) &= va + v \\ f(-1) &= 3a - 1 \end{aligned} \right\} 12a + 12 = 3a - 1 \rightarrow 10a = -13 \rightarrow a = -1,3$$

$$\left. \begin{aligned} 2 - \sqrt{3} &= \left(\frac{\sqrt{3}}{\sqrt{4}} + \frac{1}{\sqrt{4}} \right)^2 \\ 2 + \sqrt{3} &= \left(\frac{\sqrt{3}}{\sqrt{4}} + \frac{1}{\sqrt{4}} \right)^2 \end{aligned} \right\} \begin{aligned} f(2 - \sqrt{3}) + f(2 + \sqrt{3}) &= \frac{\sqrt{3} - 1}{\sqrt{4}} + \frac{\sqrt{3}}{\sqrt{3} - 1} + 2 + \frac{\sqrt{3} + 1}{\sqrt{4}} \\ &+ \frac{\sqrt{3}}{\sqrt{3} + 1} + 2 = \frac{3 - 2\sqrt{3} + 2}{\sqrt{4} - \sqrt{3}} + \frac{3 + 2\sqrt{3} + 2}{\sqrt{4} + \sqrt{3}} + 4 = \\ &\frac{\sqrt{34} - \sqrt{12}}{\sqrt{4} - \sqrt{3}} + \frac{\sqrt{34} + \sqrt{12}}{\sqrt{4} + \sqrt{3}} + 4 = \frac{\sqrt{12}(\sqrt{3} - 1)}{\sqrt{3}(\sqrt{3} - 1)} + \frac{\sqrt{12}(\sqrt{3} + 1)}{\sqrt{3}(\sqrt{3} + 1)} + 4 = 2\sqrt{4} + 4 \end{aligned}$$

$$\begin{aligned} 2f(x) - 3f(-x) &= 4x^2 - x \cdot x^2 \rightarrow 4f(x) - 9f(-x) = 4x^2 - 2x \\ 2f(-x) - 3f(x) &= 4x^2 + x \cdot x^2 \rightarrow 4f(-x) - 9f(x) = 12x^2 + 3x \\ -5f(x) &= 10x^2 + x \rightarrow f(x) = -2x^2 - \frac{x}{5} \end{aligned}$$

$$\left. \begin{aligned} x=0 &\rightarrow 2f(0) = 3m - 1 \\ x=-2 &\rightarrow 4f(0) = 1\omega + \omega m \end{aligned} \right\} 9m - 3 = 1\omega + \omega m \rightarrow m = 1, \omega$$

$$f(0) = \frac{3m - 1}{2} = 1, \omega$$

$$f(x) = ax + b \rightarrow f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = \frac{ax^2 + bx}{x} + \frac{a + bx}{x} = \frac{3x^2 - 12x + 3}{x}$$

$$\rightarrow a = 3, b = -4 \rightarrow f(x) = 3x - 4 \rightarrow f(-1) = -7$$