

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$

$\Rightarrow \frac{2x-1}{x(x-1)} \leq 0$ $\rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{x}{x-1} + \frac{1}{x+2}} = \frac{\frac{x-2x-2}{x(x+1)}}{\frac{x^2+x+2x+2}{(x-1)(x+2)}} = \frac{-x-2}{\frac{x^2+3x+2}{(x-1)(x+2)}} \rightarrow D_f = \mathbb{R} - \{-2, -1, 0, \frac{2}{3}\}$

الف) $f(x) = \sqrt{(\frac{1}{x}-9)(32-x^2)} \rightarrow (\frac{1}{x}-9)(32-x^2) \geq 0$

$\rightarrow D_f = (-\infty, -2] \cup [\frac{1}{9}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1}$

$\Rightarrow \begin{cases} \text{I) } x-1 \geq 0 \Rightarrow x \geq 1 \\ \text{II) } 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10 \end{cases} \Rightarrow \begin{cases} x \geq 1 \\ x \leq 10 \end{cases} \Rightarrow D_f = [1, 10]$

$f(x) = \frac{\log_2(x^2-x-2)}{\sqrt{x^2-1}+1} \Rightarrow \begin{cases} \text{I) } x^2-x-2 \geq 0 \Rightarrow (x-2)(x+1) \geq 0 \Rightarrow x \leq -1 \text{ or } x \geq 2 \\ \text{II) } x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \leq -1 \text{ or } x \geq 1 \\ \text{III) } \sqrt{x^2-1}+1 \neq 0 \Rightarrow \sqrt{x^2-1} \neq -1 \end{cases}$

$(\text{I} \cap \text{II}) \cap (\text{III}) \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$D_f = [a, b]$
ریشه های معادله $x_1 = -2$ و $x_2 = b$ باشند پس $-x^2 + ax + 3 \geq 0 \Rightarrow$
به آنلی ریشه ها باید عبارت برابر با صفر شود

$\Rightarrow x_1 = -2 \Rightarrow -4 - 2a + 3 = 0 \Rightarrow a = -\frac{1}{2}$ (I)

$a = x_1 + x_2 \Rightarrow -\frac{1}{2} = -2 + b \Rightarrow b = \frac{3}{2}$ (II)

$(\text{I}) \cap (\text{II}) \Rightarrow a + b = 1$

$g(x) = \sqrt{f(x)-x} \Rightarrow f(x)-x \geq 0$
 $f(x) = \begin{cases} 2x-2; & x \geq 1 \\ 2x+3; & x < 1 \end{cases} \Rightarrow f(x)-x = \begin{cases} x-2; & x \geq 1 \\ x+3; & x < 1 \end{cases} \Rightarrow \begin{cases} x-2 \geq 0 \Rightarrow x \geq 2 \\ x+3 \geq 0 \Rightarrow x \geq -3 \end{cases}$

$(\text{I}) \cup (\text{II}) \Rightarrow -3 \leq x \Rightarrow D_f = [-3, +\infty)$

$$2f(2) = f(-2) + a \Rightarrow 2x(\frac{a+1}{x+1}) = 3a - 4 + a \Rightarrow 12a + 12 = 4a - 4$$

$$\Rightarrow 10a = -16 \Rightarrow \boxed{a = -1.6}$$

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یادداشت: $f(x) = \sqrt{x + \frac{1}{x}} + 2 = \sqrt{x + \frac{1}{x} + 2} + 2 \Rightarrow f(x) = \sqrt{x + \frac{1}{x} + 2} + 2$

$(2 + \sqrt{3}) \times (2 - \sqrt{3}) = 1 \Rightarrow$ معکوس یکدیگرند و با توجه به افتادار $2 + \sqrt{3}$ و $2 - \sqrt{3}$ تابع f همواره از $\frac{1}{a}$ مقدار یکسانی می‌دهد پس:

$$f(2 + \sqrt{3}) + f(2 - \sqrt{3}) = 2f(2 + \sqrt{3})$$

$$f(2 + \sqrt{3}) = \sqrt{2 + \sqrt{3} + \frac{1}{2 + \sqrt{3}}} + 2 = \sqrt{2 + \sqrt{3} + 2 - \sqrt{3}} + 2 + 2 = 2 + \sqrt{3}$$

$$\Rightarrow 2f(2 + \sqrt{3}) = \boxed{4 + 2\sqrt{3}}$$

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$$\begin{cases} 2f(x) - 3f(-x) = 4x^2 - x \\ 2f(-x) - 3f(x) = 4x^2 + x \end{cases} \rightarrow \begin{cases} 4f(x) - 6f(-x) = 8x^2 - 2x \\ 6f(-x) - 9f(x) = 12x^2 + 3x \\ \hline -5f(x) = 20x^2 + x \end{cases}$$

$$\Rightarrow \boxed{f(x) = -4x^2 - \frac{1}{5}x}$$

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$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1$$

$$\begin{cases} x=-2 \Rightarrow f(0) = 16 + 2m + 3m - 1 = 5m + 15 \\ x=0 \Rightarrow 2f(0) = 3m - 1 \Rightarrow f(0) = \frac{3m-1}{2} \end{cases}$$

$$\Rightarrow 5m + 15 = \frac{3m-1}{2} \Rightarrow 10m + 30 = 3m - 1 \Rightarrow 7m = -31 \Rightarrow m = -\frac{31}{7}$$

$$\Rightarrow f(0) = \frac{3(-\frac{31}{7}) - 1}{2} = \frac{-\frac{93}{7} - 1}{2} = \frac{-\frac{100}{7}}{2} = -\frac{50}{7}$$

$$\Rightarrow \boxed{f(0) = -\frac{50}{7}}$$

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$$x=-1 \Rightarrow f(-1) + f(-1) = \frac{f(1) - (f(-1) + 3)}{-1}$$

$$\Rightarrow 2f(-1) = \frac{1}{-1} \Rightarrow \boxed{f(-1) = -\frac{1}{2}}$$

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