

الف) $\sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \Rightarrow -\frac{1}{x} + \frac{1}{x-1} \Rightarrow D_f = (0, 1) \cup [\frac{1}{2}, 1)$ (۱)

ب) $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{-x+1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{-x^2+1-x^2}{x(x-1)} \geq 0 \Rightarrow \frac{-2x^2+1}{x(x-1)} \geq 0$
 $\frac{-x^2-2x-1}{x^2+x} \geq -\frac{1}{2} \Rightarrow \frac{1}{x+1} \Rightarrow x \neq -1 \Rightarrow \mathbb{R} - \{-2, -1, 0, 1, -\frac{1}{2}\}$
 $\frac{-x^2-2x-1}{x^2+x-2} \geq -\frac{1}{2} \Rightarrow \frac{1}{x+1} \Rightarrow x \neq -1$

الف) $\left(\frac{1}{x}\right)^n = 9 \Rightarrow \frac{1}{x^n} = 9 \Rightarrow x^n = \frac{1}{9} \Rightarrow x = \pm \frac{1}{3}$
 $x = \pm \frac{1}{3} \Rightarrow D_f = (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, +\infty)$ ✓

ب) $\sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow \sqrt{x-1} \geq 0 \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow 1 \leq x \leq 10$
 $D_f = [1, 10]$ ✓

$f(x) = \frac{\log(x^2 - x - 4)}{\sqrt{x^2 - 1} + 1} \Rightarrow x^2 - x - 4 > 0 \Rightarrow (x+1)(x-4) > 0 \Rightarrow x < -1 \text{ or } x > 4$
 $\sqrt{x^2 - 1} \geq 0 \Rightarrow x \geq 1 \text{ or } x \leq -1$
 $D_f = \mathbb{R} - [-1, 4]$ ✓

$\sqrt{c + 2x - x^2} \xrightarrow{x=-2} \sqrt{-c + 4 - 4} = 0 \Rightarrow -c + 4 - 4 = 0 \Rightarrow -c = 0 \Rightarrow c = 0$
 $\Rightarrow -x^2 - \frac{x}{2} + c = 0 \Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4(-1)c}}{2(-1)} = -2, \frac{c}{2}$
 $D_f = [-c, 0]$
 $c + b = 0 \Rightarrow c = -b$ ✓

$g(x) = x > 1 \Rightarrow \sqrt{2x - c - x} \rightarrow \sqrt{x - c} \rightarrow 2x - c > 0 \Rightarrow x > \frac{c}{2}$
 $x > 1 \Rightarrow I$
 $\Rightarrow I \cap I \Rightarrow D_g = [-c, +\infty)$ ✓
 $h(x) = x < 1 \Rightarrow \sqrt{2x + c - x} \rightarrow \sqrt{x + c} \rightarrow x + c > 0 \Rightarrow x > -c$
 $x < 1 \Rightarrow II$
 $\Rightarrow D_h = [-c, 1)$

$$f(0) = f(1) + a \Rightarrow (a+1)(1+d) = 2a - \varepsilon + a$$

$$1 \cdot 2a + 1 \cdot \varepsilon = 3a - \varepsilon$$

$$1 \cdot 2a = 2a$$

$$a = 1, 1 \checkmark$$

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$$\sqrt{x-\sqrt{c}} + \frac{1}{\sqrt{x-\sqrt{c}}} + c + \sqrt{x+\sqrt{c}} + \frac{1}{\sqrt{x+\sqrt{c}}} - c$$

$$= \frac{\sqrt{x-\sqrt{c}} + \sqrt{x+\sqrt{c}} + \sqrt{x+\sqrt{c}} + \sqrt{x-\sqrt{c}} + c + c}{\sqrt{(x-\sqrt{c})(x+\sqrt{c})}}$$

$$\Rightarrow \frac{(\sqrt{x-\sqrt{c}} + \sqrt{x+\sqrt{c}} + c) \cdot 2}{\sqrt{4}} = \varepsilon + c \checkmark$$

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$$f(x) - f(-x) = \varepsilon x^5 - x \Rightarrow A$$

$$f(-x) - f(x) = \varepsilon x^5 + x \Rightarrow B$$

$$B + A \Rightarrow f(x) - f(x) = -\omega f(x)$$

$$\Rightarrow -\omega f(x) = \varepsilon x^5 + x$$

$$f(x) = -\varepsilon x^5 - \frac{x}{\omega} \checkmark$$

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$$(x+c)f(x) - cxf(x+c) = \varepsilon x^5 - mx + c^{m-1}$$

$$\left(\begin{array}{l} x=0 \\ x=c \end{array} \right) \Rightarrow f(0) = c^{m-1} \Rightarrow 4f(0) = 4c^{m-1}$$

$$\Rightarrow 4c^{m-1} = 10 + 0m$$

$$m = 2, 0 \checkmark$$

$$f(0) = \frac{10-1}{4} = \frac{9}{4} \checkmark$$

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$$f(-1) \Rightarrow f(-1) + f\left(\frac{1}{-1}\right) = \frac{c + 1c + c}{-1}$$

$$f(-1) = -9 \checkmark$$

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