۲۰

نام و نام خانوادگی ......علی جمالی...... پاسخنامه تشریحی تکلیف شماره ...۴... کلاس ..یازدهم ریاضی A..........

۱ (۴)

الف) $x \neq 0 \;,\; x-1 \neq 0 \;,\; \dfrac{x-1}{x} - \dfrac{x}{x-1} \geq 0 \Rightarrow \dfrac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \Rightarrow \dfrac{-2x+1}{x(x-1)} \geq 0$

x: $0 \quad \frac{1}{2} \quad 1$; $+ \,\|\, - \,0\, + \,\|\, -$ $\Rightarrow \boxed{D_f = (-\infty, 0) \cup [\tfrac{1}{2}, 1)}$ ✓

ب) $x+1 \neq 0$ $\boxed{x \neq -1}$, $\boxed{x \neq 0}$, $x-1 \neq 0$ $\boxed{x \neq 1}$, $x+2 \neq 0$ $\boxed{x \neq -2}$, $\dfrac{3}{x-1} + \dfrac{1}{x+2} \neq 0 \Rightarrow \dfrac{3(x+2) + x-1}{(x-1)(x+2)} \neq 0$

$\Rightarrow \dfrac{4x+5}{(x-1)(x+2)} \neq 0 \Rightarrow \boxed{x = \frac{-5}{4}}$

$\boxed{D_f = \mathbb{R} - \{-2, \frac{-5}{4}, -1, 0, 1\}}$ ✓

۲ (۴)

الف) $\left((\tfrac{1}{3})^x - 9\right)(32 - 4^x) \geq 0 \Rightarrow (3^{-x} - 3^2)(2^5 - 2^{2x}) \geq 0 \qquad 3>1 \Rightarrow 3^\alpha \geq 3^\beta \Leftrightarrow \alpha \geq \beta$

$\Rightarrow$ x: $-2 \quad \frac{5}{2}$; $+ \,0\, - \,0\, +$ $\Rightarrow \boxed{D_f = (-\infty, -2] \cup [\tfrac{5}{2}, +\infty)}$ ✓

ب) $x-1 \geq 0 \Rightarrow x \geq 1$

$\sqrt{y+1} \geq 0 \Rightarrow \sqrt[4]{x-1} \leq 3 \Rightarrow x-1 \leq 3^4 = 81 \Rightarrow x \leq 82$ $\Big\}$ $1 \leq x \leq 82 \Rightarrow \boxed{D_f = [1, 82]}$ ✓

۳ (۴)

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0 \Rightarrow$ x: $-1 \quad 2$; $+ \,0\, - \,0\, +$ (I) $x \in (-\infty, -1) \cup (2, +\infty)$

$x^2 - 1 \geq 0$

$(x-1)(x+1) \geq 0 \Rightarrow$ x: $-1 \quad 1$; $+ \,0\, - \,0\, +$ (II) $x \in (-\infty, -1] \cup [1, +\infty)$

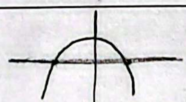
$\sqrt{x^2-1} + 1 \neq 0 \Rightarrow \sqrt{x^2-1} \neq -1$, $\sqrt{a} \geq 0$ $\Rightarrow x \in \mathbb{R}$ (III)

$I \cap II \cap III = (-\infty, -1) \cup (2, +\infty)$

$\boxed{D_f = (-\infty, -1) \cup (2, +\infty)}$ ✓

۴ (۴)

$f(x) = -x^2 + ax + 3 \geq 0$



$\Rightarrow D_f = [x_1, x_2] \;,\; f(x_1) = f(x_2) = 0 \;,\; x_1 < x_2$

$\Rightarrow f(-2) = 0 \Rightarrow -(-2)^2 + a(-2) + 3 = 0 \Rightarrow -4 - 2a + 3 = 0 \Rightarrow 2a = -1 \Rightarrow \boxed{a = \frac{-1}{2}}$

$f(b) = 0 \Rightarrow P = b \cdot (-2) = \dfrac{c}{a} = \dfrac{3}{-1} = -3 \Rightarrow \boxed{b = \frac{3}{2}}$

$a + b = \dfrac{-1}{2} + \dfrac{3}{2} = \boxed{1}$ ✓

۵ (۴)

$f(x) \geq x$

① $x \geq 1 \Rightarrow f(x) = 3x - 2 \geq x \Rightarrow 2x \geq 2 \Rightarrow \left.\begin{matrix} x \geq 1 \\ ① \; x \geq 1 \end{matrix}\right\}$ $\boxed{x \geq 1}$ I

② $x \leq 1 \Rightarrow f(x) = 2x + 3 \geq x \Rightarrow \left.\begin{matrix} x \geq -3 \\ ② \; x \leq 1 \end{matrix}\right\}$ $\boxed{-3 \leq x \leq 1}$ II

$\boxed{D_f = I \cup II = [-3, +\infty)}$ ✓

$5>1 \Rightarrow f(5) = (a+1)(5+2) = 7a+7 \Rightarrow \boxed{2f(5) = 14a+14}$

$-2<1 \Rightarrow f(-2) = 3a+2(-2) = \boxed{3a-4}$

$2f(5) = f(-2)+a \Rightarrow 14a+14 = 3a-4+a = 4a-4$

$\Rightarrow 10a = -18 \Rightarrow \boxed{a = -1.8}$

2 — 6

$\frac{1}{2+\sqrt{3}} = \frac{1(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = 2-\sqrt{3}$

$\Rightarrow \frac{1}{2-\sqrt{3}} = 2+\sqrt{3}$

$\frac{1}{\sqrt{x}} = \frac{\sqrt{1}}{\sqrt{x}} = \sqrt{\frac{1}{x}}$

$\Rightarrow f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2\left(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 2\right) =$

$4 + 2\left(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}\right) = 4 + 2\left(\sqrt{\left(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}\right)^2}\right) =$

$4 + 2\left(\sqrt{2+\sqrt{3}+2-\sqrt{3}+2\underbrace{\sqrt{(2+\sqrt{3})(2-\sqrt{3})}}_{1}}\right) = \boxed{4+2\sqrt{6}}$

2 — 7

$2f(x) - 3f(-x) = 4x^2 - x \quad : P(x)$

$P(-x) \Rightarrow 2f(-x) - 3f(x) = 4(-x)^2 - (-x) = 4x^2 + x$

$P(x), P(-x) \Rightarrow \left.\begin{array}{l} 4f(x) - 6f(-x) = 8x^2 - 2x \\ 6f(-x) - 9f(x) = 12x^2 + 3x \end{array}\right\} \Rightarrow -5f(x) = 20x^2 + x$

$\Rightarrow \boxed{f(x) = -4x^2 - \frac{1}{5}x}$

2 — 8

$\left.\begin{array}{l} P(0) \Rightarrow 2f(0) = 3m-1 \Rightarrow 6f(0) = 9m-3 \\ P(-2) \Rightarrow 6f(0) = 4(-2)^2 - m(-2) + 3m - 1 = 15+5m \end{array}\right\} \quad 9m-3 = 15+5m$

$4m = 18$

$\boxed{m = 4.5}$

$\Rightarrow 2f(0) = 3m-1 = 3\times 4.5 - 1 = 12.5$

$\Rightarrow \boxed{f(0) = 6.25}$

4 — 9

$\left.\begin{array}{l} f(x) = ax+b \\ \Rightarrow f(\frac{1}{x}) = \frac{a}{x}+b \end{array}\right\} \Rightarrow f(x) + f(\frac{1}{x}) = a\underbrace{\left(x+\frac{1}{x}\right)}_{\frac{x^2+1}{x}} + 2b = \frac{ax^2+2bx+a}{x} = \frac{3x^2-12x+3}{x}$

$\Rightarrow \boxed{a=3}$

$2b = -12 \Rightarrow \boxed{b=-6}$

$f(x) = ax+b = 3x-6$

$f(-1) = 3(-1)-6 = \boxed{-9}$

2 — 10