

الف)  $f(m) = \sqrt{\frac{m-1}{m} - \frac{m}{m-1}}$   $\xrightarrow{D_f}$   $\begin{cases} \text{ریشه منجم} \neq 0 \\ \text{و} > 0 \end{cases} \rightarrow \begin{cases} \text{I} \begin{cases} m+0 \\ m \neq 1 \end{cases} \\ \text{II} \begin{cases} \frac{m-1}{m} - \frac{m}{m-1} > 0 \end{cases} \end{cases}$

$\Rightarrow \frac{-2m+1}{m(m-1)} > 0 \xrightarrow{\text{ریشه}} \begin{cases} m = \frac{1}{2} \\ m = 0 \\ m = 1 \end{cases} \rightarrow \begin{array}{c} 0 \quad \frac{1}{2} \quad 1 \\ + \quad - \quad + \quad - \end{array} \xrightarrow{\text{اشتراک}} \begin{array}{c} 0 \quad \frac{1}{2} \quad 1 \\ + \quad - \quad + \end{array}$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$  ✓

ب)  $f(m) = \frac{1}{m+1} - \frac{1}{m} \xrightarrow{\text{تکلیف}} \begin{cases} \text{ریشه منجم} \neq 0 \end{cases} \rightarrow \begin{cases} m+1 \neq 0 \rightarrow m \neq -1 \\ m \neq 0 \rightarrow m \neq 0 \\ m-1 \neq 0 \rightarrow m \neq 1 \\ m+2 \neq 0 \rightarrow m \neq -2 \end{cases}$

$\frac{1}{m-1} + \frac{1}{m+2} \neq 0 \rightarrow \frac{m+2}{(m-1)(m+2)} \neq 0 \rightarrow \frac{m+5}{(m-1)(m+2)} \neq 0 \rightarrow \frac{m+5}{(m-1)(m+2)} \neq 0 \rightarrow m \neq -\frac{5}{2}$

$\Rightarrow \frac{m+5}{(m-1)(m+2)} \neq 0 \rightarrow m+5 \neq 0 \rightarrow m \neq -5$   $\xrightarrow{\text{اشتراک}} D_f = \mathbb{R} - \{-2, -\frac{5}{2}, -1, 0, +1\}$  ✓

الف)  $f(m) = \sqrt{((\frac{1}{2})^m - 9)(32 - 4^m)}$   $\xrightarrow{\text{ریشه منجم}} \begin{cases} (\frac{1}{2})^m - 9 > 0 \\ (32 - 4^m) > 0 \end{cases}$

$\Rightarrow \begin{array}{c} -2 \quad +\frac{5}{2} \\ + \quad - \quad + \end{array} \xrightarrow{\text{تعیین علامت}} \begin{array}{c} m = -2 \quad \leftarrow \quad \rightarrow \quad m = \frac{5}{2} \end{array}$

$D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$  ✓

ب)  $\sqrt{m-1} + \sqrt{y+1} = 3 \xrightarrow{\text{شرایط}} \begin{cases} m-1 > 0 \rightarrow m > 1 \\ y+1 > 0 \rightarrow y > -1 \end{cases} \rightarrow \sqrt{y+1} = 3 - \sqrt{m-1} \rightarrow 3 > \sqrt{m-1}$

$9 > \sqrt{m-1} \xrightarrow{\text{توان}} 81 > m-1 \xrightarrow{\text{اشتراک}} \begin{cases} m > 1 \\ m \leq 82 \end{cases}$

$D_f = [1, 82]$  ✓

الف)  $f(m) = \frac{\log(m^2 - m - 2)}{\sqrt{m^2 - 1} + 1}$   $\xrightarrow{\text{تکلیف}} \begin{cases} \text{ریشه منجم} \neq 0 \\ \text{و} > 0 \end{cases} \rightarrow \begin{cases} m^2 - 1 = (m-1)(m+1) > 0 \\ m^2 - m - 2 = (m-2)(m+1) > 0 \end{cases}$

$\Rightarrow \begin{array}{c} -1 \quad +2 \\ + \quad - \quad + \end{array} \xrightarrow{\text{اشتراک}} \begin{array}{c} -1 \quad +1 \quad +2 \\ + \quad - \quad + \end{array} \xrightarrow{\text{اشتراک}} D_f = (-\infty, -1) \cup (2, +\infty)$  ✓

سوال ۴

۱۵

$$f(m) = \sqrt{3 + am - m^2} \rightarrow \phi = [-r, b]$$

$$a + b = ?$$

رابطه را در  $x = -r$  قرار می‌دهیم

$$-(-r)^2 + a(-r) + 3 = -r^2 - ar + 3 = -r^2 - \frac{1}{4}r + 3 = 0$$

$$\Rightarrow -r^2 - \frac{1}{4}r + 3 = 0 \rightarrow r^2 + \frac{1}{4}r - 3 = 0$$

$$r = \frac{-\frac{1}{4} \pm \sqrt{\frac{1}{16} + 12}}{2} = \frac{-\frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{192}{16}}}{2} = \frac{-\frac{1}{4} \pm \sqrt{\frac{193}{16}}}{2}$$

$$r = \frac{-\frac{1}{4} \pm \frac{\sqrt{193}}{4}}{2} = \frac{-\frac{1}{4} \pm \frac{\sqrt{193}}{4}}{2}$$

اما  $r = -\frac{1}{4}$  را می‌گیریم

$$a = -\frac{1}{4}$$

حالا  $b$  را پیدا می‌کنیم

$$b = +\frac{3}{4}$$

پس  $a + b = -\frac{1}{4} + \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

سوال ۵

۲

$$f(m) = \begin{cases} (a+1)(m+2) & m \geq 1 \\ 2m+3 & m < 1 \end{cases}$$

$$g(m) = \sqrt{f(m) - m}$$

$$f(m) - m = \begin{cases} (a+1)(m+2) - m & m \geq 1 \\ 2m+3 - m & m < 1 \end{cases} = \begin{cases} (a+1)m + 2(a+1) - m & m \geq 1 \\ m+3 & m < 1 \end{cases}$$

$$= \begin{cases} am + 2(a+1) & m \geq 1 \\ m+3 & m < 1 \end{cases}$$

برای اینکه نمودار شوم این تابع در  $m=1$  پیوسته باشد

پس  $f(1) = g(1)$

$$(a+1)(1+2) = \sqrt{a(1) + 2(a+1)}$$

$$3(a+1) = \sqrt{a + 2a + 2} = \sqrt{3a + 2}$$

$$9(a+1)^2 = 3a + 2$$

$$9(a^2 + 2a + 1) = 3a + 2$$

$$9a^2 + 18a + 9 = 3a + 2$$

$$9a^2 + 15a + 7 = 0$$

این معادله را حل می‌کنیم

$$a = \frac{-15 \pm \sqrt{225 - 252}}{18}$$

پس جواب  $a = -\frac{1}{3}$  است

سوال ۶

۲

$$f(m) = \begin{cases} (a+1)(m+2) & m \geq 1 \\ 2a+2m & m < 1 \end{cases}$$

$$f(a) = f(-r) + a$$

$$a = ? \rightarrow 2(a+1)(r) = 2a + 2(-r) + a$$

$$2a + 2r = 2a - 2r + a$$

$$4r = a \Rightarrow a = 4r$$

اما  $r = -\frac{1}{4}$  را می‌گیریم

$$a = -1$$

سوال ۷

۲

$$f(m) = \sqrt{m} + \frac{1}{\sqrt{m}} + 2$$

$$f(r-\sqrt{3}) + f(r+\sqrt{3}) = ?$$

$$f(r-\sqrt{3}) = \sqrt{r-\sqrt{3}} + \frac{1}{\sqrt{r-\sqrt{3}}} + 2$$

$$f(r+\sqrt{3}) = \sqrt{r+\sqrt{3}} + \frac{1}{\sqrt{r+\sqrt{3}}} + 2$$

$$\Rightarrow f(r-\sqrt{3}) + f(r+\sqrt{3}) = \sqrt{r-\sqrt{3}} + \sqrt{r+\sqrt{3}} + \frac{1}{\sqrt{r-\sqrt{3}}} + \frac{1}{\sqrt{r+\sqrt{3}}} + 4$$

$$A = \sqrt{r-\sqrt{3}} + \sqrt{r+\sqrt{3}}$$

$$A^2 = r - \sqrt{3} + r + \sqrt{3} + 2\sqrt{(r-\sqrt{3})(r+\sqrt{3})} = 2r + 2\sqrt{r^2 - 3}$$

$$A^2 = 2r + 2\sqrt{r^2 - 3} = 4 + 2\sqrt{r^2 - 3}$$

$$A = \sqrt{4 + 2\sqrt{r^2 - 3}}$$

سوال ۸

۲

$$2f(m) - 3f(-m) = 4m^2 - m$$

$$f(m) = ?$$

$$\begin{cases} x \rightarrow 2f(m) - 3f(-m) = 4m^2 - m \\ -m \rightarrow 2f(-m) - 3f(m) = 4m^2 + m \end{cases}$$

$$\begin{cases} 2f(m) - 3f(-m) = 4m^2 - m \\ 2f(-m) - 3f(m) = 4m^2 + m \end{cases}$$

$$-5f(m) = 20m^2 + m$$

$$f(m) = \frac{20m^2 + m}{-5} = -4m^2 - \frac{1}{5}m = f(m)$$

$$(n+2)f(n) - 3nf(n+2) = 5n^2 - mn + 3m-1$$

$$f(0) = ?$$

سوال 9  
✓

$$\left\{ \begin{array}{l} n=0 \rightarrow 2f(0) - 3(0)f(2) = 5(0)^2 - m(0) + 3m-1 \Rightarrow 2f(0) = 3m-1 \\ \Rightarrow f(0) = \frac{3m-1}{2} \end{array} \right.$$

$$\left\{ \begin{array}{l} m=-1 \rightarrow (-2+2)f(-1) - 3(-1)f(0) = 5(-1)^2 - m(-1) + 3m-1 \\ \quad + 4f(0) = 14 + 2m + 3m-1 = 15 + 5m \rightarrow f(0) = \frac{15+5m}{4} = \frac{3m-1}{2} = 9m-3 = 5m+12 \end{array} \right.$$

$$\Rightarrow 5m=12 \rightarrow m = \frac{12}{5} = \frac{4}{1} = 4, 2 \checkmark \Rightarrow f(0) = \frac{3m-1}{2} = \frac{3(4,2)-1}{2} = \frac{12,6-1}{2} = 4,25 \checkmark$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{5x^2 - 12x + 3}{x} \xrightarrow{-1 \text{ ليحذف}} 2f(-1) + f(-1) = \frac{5(-1)^2 - 12(-1) + 3}{-1} = \frac{5+12+3}{-1} = -20 \leftarrow \text{سوال 10}$$

$$\Rightarrow 2f(-1) = -20 \rightarrow f(-1) = -10 \checkmark$$