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الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \to \frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{1-2x}{x(x-1)}\geq 0$

$\xrightarrow{x\neq 0,1}$ $\begin{array}{c|c|c|c} 0 & \frac{1}{2} & 1 & \\ + \quad - & + & - & \end{array} \longrightarrow D_f=(-\infty,0)\cup[\frac{1}{2},1)$ ✓

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \longrightarrow x\neq -1,0,1,-2 \xrightarrow{\text{مخرج}} \frac{3}{x-1}+\frac{1}{x+2}=\frac{4x+5}{(x-1)(x+2)}$, $x\neq -\frac{5}{4}$ $\Rightarrow D_f=\mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}$ ✓

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الف) $f(x)=\sqrt{\left((\frac{1}{3})^x-9\right)(32-4^x)} \to \left((\frac{1}{3})^x-9\right)(32-4^x)\geq 0$

$\begin{cases}(\frac{1}{3})^x-9=0 \to x=-2\\ 32-4^x=0 \to 4^x=32 \to x=\frac{5}{2}\end{cases}$

$\longrightarrow \begin{array}{c|c|c} -2 & & \frac{5}{2} \\ + & - & + \end{array} \Rightarrow D_f=(-\infty,-2]\cup[\frac{5}{2},+\infty)$ ✓

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \Rightarrow x-1\geq 0 \to x\geq 1$ (1) $\Rightarrow \sqrt{y+1}=3-\sqrt[4]{x-1} \xrightarrow{y+1\geq 0} 3-\sqrt[4]{x-1}\geq 0$

$y+1\geq 0 \to y\geq -1 \longrightarrow \sqrt[4]{x-1}\leq 3 \to x\leq 82$ (2)

(1) $\cap$ (2) $\to D_f=[1,82]$ ✓

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$f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$ $\quad x^2-x-2>0 \to (x-2)(x+1)>0 \to \begin{array}{c|c|c} -1 & & 2 \\ + & - & + \end{array}$ (1)

$\Rightarrow x^2-1\geq 0 \Rightarrow x^2\geq 1 \Rightarrow x>1 , x<-1$ (2)

$\sqrt{x^2-1}+1\neq 0 \to \sqrt{x^2-1}\neq -1$ ✓

(1) $\cap$ (2) $\to D_f=(-\infty,-1)\cup(2,+\infty)$ ✓

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$f(x)=\sqrt{3+ax-x^2}$ $\quad D_f=[-2,b]$

$\Rightarrow 3+ax-x^2\geq 0 \to -x^2+ax+3=-(x+2)(x-b) \Rightarrow -(x+2)(x-b)=-x^2+(b-2)x+2b$

$2b=3 \to \boxed{b=\frac{3}{2}}$

$a=b-2 \to \boxed{a=\frac{3}{2}-2=-\frac{1}{2}}$

$a+b=-\frac{1}{2}+\frac{3}{2}=1$ ✓

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$f(x)=\begin{cases}3x-2 ; & x\geq 1\\ 2x+3 ; & x<1\end{cases}$ $\qquad g(x)=\sqrt{f(x)-x}$

$\xrightarrow{x\geq 1} f(x)-x=3x-2-x=2x-2=2(x-1) \longrightarrow 2(x-1)\geq 0 \Rightarrow [1,+\infty)$ (1)

$\xrightarrow{x<1} f(x)-x=2x+3-x=x+3 \longrightarrow x+3\geq 0 \Rightarrow x\geq -3 \to [-3,+\infty) \to [-3,1)$ (2)

$\xrightarrow{(1)\cup(2)} D_g=[-3,\infty)$ ✓

$$f(x)=\begin{cases}(a+1)(x+2)\ ; & x>1\\ 3a+2x\ ; & x\leq 1\end{cases} \qquad 2f(5)=f(-2)+a$$

$$\Longrightarrow 2(7a+7)=3a-4+a$$ (2) 6

$$\longrightarrow 14a+14=4a-4 \to 10a=-18 \to \boxed{a=-\frac{18}{10}=-\frac{9}{5}}$$

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \qquad f(2-\sqrt{3})+f(2+\sqrt{3})=?$$ (2)

$$\Longrightarrow f(2-\sqrt{3})+f(2+\sqrt{3})=\Big(\underbrace{\sqrt{2-\sqrt{3}}}_{a}+\frac{1}{\sqrt{2-\sqrt{3}}}\Big)+\Big(\underbrace{\sqrt{2+\sqrt{3}}}_{b}+\frac{1}{\sqrt{2+\sqrt{3}}}\Big)+4$$ 7

$$a \to a\times b=1 \leftarrow b \qquad (a+b)^2=6 \to a+b=\sqrt{6}$$

$$\Big(a+\frac{1}{a}\Big)+\Big(b+\frac{1}{b}\Big)+4=(b+a)+(b+a)+4=2(a+b)+4 \Longrightarrow \boxed{4+2\sqrt{6}}$$

$$2f(x)-3f(-x)=4x^2-x \xrightarrow{\times 2} \begin{cases}4f(x)-6f(-x)=8x^2-2x\\ 6f(-x)-9f(x)=12x^2+3x\end{cases}$$ (2)

$$\xrightarrow{x=-x} 2f(-x)-3f(x)=4x^2+x \xrightarrow{\times 3}$$ 8

$$+ \quad -5f(x)=20x^2+x$$

$$\Longrightarrow \boxed{f(x)=-4x^2-\frac{1}{5}x}$$

$$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1$$ (2)

$$x=-2 \longrightarrow 6f(0)=16+2m+3m-1 \to 6f(0)=15+5m \xRightarrow{f(0)=\frac{3m-1}{2}} 9m-3=15+5m$$ 9

$$x=0 \longrightarrow 2f(0)=3m-1 \to f(0)=\frac{3m-1}{2} \qquad 4m=18 \to m=\frac{18}{4}=\frac{9}{2}$$

$$\Longrightarrow f(0)=\frac{3m-1}{2} \xrightarrow{m=\frac{9}{2}} \frac{\frac{27}{2}-1}{2}=\frac{\frac{25}{2}}{\frac{2}{1}}=\frac{25}{4} \longrightarrow \boxed{f(0)=\frac{25}{4}=6.25}$$

$$f(x)+f\Big(\frac{1}{x}\Big)=\frac{3x^2-12x+3}{x}$$ (2)

$$\begin{matrix}f(x)=ax+b\\ f(\frac{1}{x})=\frac{a}{x}+b\end{matrix} \longrightarrow f(x)+f\Big(\frac{1}{x}\Big)=ax+b+\frac{a}{x}+b=ax+\frac{a}{x}+2b=\frac{3x^2-12x+3}{x}$$ 10

$$\frac{ax^2+2bx+a}{x}=\frac{3x^2-12x+3}{x} \Longrightarrow a=3,\ b=-6,\ 3x-6=f(x) \Longrightarrow \boxed{f(-1)=-3-6=-9}$$