پاسخ مکاتبان یازدهم A

۱) $\lim_{x\to 1} \frac{4x^2-7x+3}{5x^2-8x+3} = \frac{(x-1)(4x-3)}{(x-1)(5x-3)} = \frac{4x-3}{5x-3} = \frac{1}{2}$

۲) $\lim_{x\to 0} \frac{|3x-1|-|3x+1|}{x} = \frac{-(3x-1)-(3x+1)}{x} = -\frac{6x}{x} = -6$

۳) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}-2)} = \sqrt{x}+2 = 2+2 = 4$

۴) $\lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} \times \frac{x+\sqrt{2x}}{x+\sqrt{2x}} = \frac{x^2-2x}{(x+\sqrt{2x})(2x^2-x-6)} = \frac{x(x-2)}{(x+\sqrt{2x})(x-2)(2x+3)} = \frac{2}{(4+2)(4+3)}$

$= \frac{1}{14}$

۵) $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} \times \frac{2+\sqrt{5-x}}{2+\sqrt{5-x}} = \frac{(1-x)(2+\sqrt{5-x})}{(1+\sqrt{x})(4-(5-x))} = \frac{(1-x)(2+\sqrt{5-x})}{-(1-x)(1+\sqrt{x})}$

$= -\frac{2+2}{1+1} = -2$

۶) $\lim_{x\to 4} \frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3} \times \frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4} \times \frac{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}$

$= \frac{(3x+4-16)\left(\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9\right)}{(5x+7-27)(\sqrt{3x+4}+4)} = \frac{8(9+9+9)}{5(4+4)} = \frac{81}{40}$

۷) $\lim_{x\to 1} \frac{\sqrt{2x+\sqrt{x}}-2}{\sqrt[3]{x}-1} \times \frac{\sqrt{2x+\sqrt{x}}+2}{\sqrt{2x+\sqrt{x}}+2} \times \frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$

$= \frac{(2x+\sqrt{x}-4)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}{(x-1)(\sqrt{2x+\sqrt{x}}+2)} = \frac{(2\sqrt{x}+4)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}{(\sqrt{x}+1)(\sqrt{2x+\sqrt{x}}+2)} = \frac{(2+4)(1+1+1)}{(1+1)(2+2)}$

$= \frac{6\times 3}{4\times 4} = \frac{9}{8}$

۸) $\lim_{x \to 0} \frac{1+\cos^3 x}{\sin^2 x} = \frac{(1+\cos x)(\cos^2 x - \cos x + 1)}{(1-\cos x)(1+\cos x)} = \frac{\cos^2 x - \cos x + 1}{1-\cos x} = \frac{1+1+1}{1+1} = \frac{3}{2}$

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۹) $\lim_{x \to \frac{\pi}{4}} \frac{1-\tan x}{\sin x - \cos x} = \frac{1-\frac{\sin x}{\cos x}}{\sin x - \cos x} = \frac{\frac{\cos - \sin}{\cos}}{\sin - \cos} = \frac{-1}{\cos} = -\frac{1}{\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

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۱۰) $\lim_{x \to \frac{\pi}{4}} \frac{\tan^2 x - 1}{\cos 2x} = \frac{\frac{\sin^2 x}{\cos^2 x} - 1}{\cos^2 x - \sin^2 x} = \frac{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}{\cos^2 x - \sin^2 x} = -\frac{1}{\cos^2 x} = -2$